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THRESHOLD LOGIC AND ITS APPLICATIONS

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Table 2.3.2 Number of Equivalence Classes

n	0	1	2	3	4	5		6
Switching functions of n or fewer variables	2	4	16	256	65,536	About 4.3×10^9	✓	About 1.8×10^{19}
Switching functions of exactly n variables	2	2	10	218	64,594	About 4.3×10^9	371	About 1.8×10^{19}
Threshold functions of n or fewer variables	2	4	14	104	1,882	94,572	609	15,028,13
Threshold functions of exactly n variables	2	2	8	72	1,536	86,080	615	14,487,04
P-equivalence classes of switching functions of n or fewer variables	2	4	12	80	3,984	37,333,248	3180	—
P-equivalence classes of switching functions of exactly n variables	2	2	8	68	3,904	37,329,264	3181	—
N-equivalence classes of threshold functions of n or fewer variables	2	3	6	20	150	3,287	2078	244,15
N-equivalence classes of threshold functions of exactly n variables	2	1	2	9	96	2,690	2079	226,36
N-equivalence classes of self-dual threshold functions of $n+1$ or fewer variables	1	2	4	12	81	1,684	2080	123,56
N-equivalence classes of self-dual threshold functions of exactly $n+1$ variables	1	0	1	4	46	1,322	2077	112,51
NP-equivalence classes of switching functions of n or fewer variables	2	3	6	22	402	1,228,158	616	400,507,806,843,72
NP-equivalence classes of switching functions of exactly n variables	2	1	3	16	380	1,227,756	618	400,507,805,615,57
NP-equivalence classes of unate functions of n or fewer variables	3	5	10	30	210	16,353	3182	—
NP-equivalence classes of threshold functions of n or fewer variables	2	3	5	10	27	119	617	1,113
NP-equivalence classes of threshold functions of exactly n variables	2	1	2	5	17	92	619	994
NPN-equivalence classes of switching functions of n or fewer variables	1	2	4	14	222	616,126	370	200,253,952,527,184
NPN-equivalence classes of switching functions of exactly n variables	1	1	2	10	208	615,904	1528	200,263,951,911,058
NPN-equivalence classes of unate functions of n or fewer variables	2	3	6	17	112	8,282	3183	—
NPN-equivalence classes of threshold functions of n or fewer variables	1	2	3	6	15	63	1529	567
NPN-equivalence classes of threshold functions of exactly n variables	1	1	1	3	9	48	1530	504
Self-duality equivalence classes of switching functions of $n+1$ or fewer variables	1	1	3	7	83	109,958	1531	—
Self-duality equivalence classes of switching functions of exactly $n+1$ variables	1	0	2	4	76	109,875	6688	—
Self-duality equivalence classes of threshold functions of $n+1$ or fewer variables	1	1	2	3	7	21	1532	135
Self-duality equivalence classes of threshold functions of exactly $n+1$ variables	1	0	1	1	4	14	3184	114

Note that some rows are f
four rows consists of self-
nonself-dual functions of

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4	5
65,536	About 4.3 × 10^9
64,594	About 4.3 × 10^9
1,882	94,572
1,536	86,080
3,984	37,333,248
3,904	37,329,264
150	3,287
96	2,690
81	1,684
46	1,322
402	1,228,158
380	1,227,756
210	16,353
27	119
17	92
222	616,126
208	615,904
112	8,282
15	63
9	48
83	109,958
76	109,875
7	21
4	14

Table 2.3.2 (Cont'd)

6	7	8
About 1.8 × 10^{19}	About 3.4 × 10^{38}	About 1.16 × 10^{77}
About 1.8 × 10^{19}	About 3.4 × 10^{38}	About 1.16 × 10^{77}
15,028,134	8,378,070,864	17,561,539,552,946
14,487,040	8,274,797,440	17,494,930,604,032
—	—	—
—	—	—
244,158	66,291,591	68,863,243,522
226,360	64,646,855	68,339,572,672
123,565	33,207,256	34,448,225,389
112,519	32,267,168	34,153,652,752
400,507,806,843,728	—	—
400,507,805,615,570	—	—
—	—	—
1,113	29,375	2,730,166
994	28,262	2,700,791
200,253,952,527,184	—	—
200,263,951,911,058	—	—
—	—	—
567	14,755	1,366,318
504	14,188	1,351,563
—	—	—
—	—	—
135	2,470	175,428
114	2,335	172,958

Note that some rows are for $n + 1$ variables instead of for n variables. Also each class in the last four rows consists of self-dual functions of exactly $n + 1$ variables and self-duality-equivalent nonself-dual functions of exactly n variables.

Table 7.5.1 Number of Pseudo-Threshold Functions

n, Number of Variables	1	2	3	4	5
Number of positive functions of n or fewer variables	3	6	20	168	7581
Number of representative[a] positive functions of n or fewer variables	3	5	10	30	210
Number of representative[a] positive pseudo-threshold functions of n or fewer variables	3	5	10	30	198
Number of representative[a] positive threshold functions of n or fewer variables	3	5	10	27	119

[a] Representatives of the class of functions that are equivalent by the permutation of variables.

[Fagerlin (University of Illinois.) 68].) Functions with five or fewer variables which are not pseudo-threshold functions are shown in the appendix.

C. R. Baugh studied further properties of pseudo-threshold functions, on the basis of which he obtained bounds on the number of positive functions [Baugh (University of Illinois) 70]. Self-dual pseudo-threshold functions were studied by Breeding [Breeding (University of Illinois) 67].

If f is self-dual, we may set $w_0 = 0$ and the total input weight is $\sum_{i=1}^{n} w_i$. Therefore we have (9.3.2.9).

If f is not self-dual, we need to find a bound on w_0 in order to get a bound on $\sum_{i=0}^{n} w_i$. Let us self-dualize f as $fx_0 \vee f^d$, if $f \supset f^d$, or as $f^d x_0 \vee f$ otherwise. Then substitute the value 1 (or 0) into x_n of this self-dualized function. It is easy to show that the function with $x_n \rightarrow 1$ is of exactly n variables. Therefore we have a linearly independent set of $n + 1$ equalities corresponding to (9.3.2.4) in terms of $w_0, w_1, \ldots, w_{n-1}, T$. Solving this, we have

$$w_0 = \frac{\Delta_0'}{\Delta'} \qquad (9.3.2.12)\dagger$$

[$w_1, \ldots, w_{n-1}$ must be identical to those given by (9.3.2.5)]. Again by the same evaluation technique we get

$$w_0 \leq 2\left(\frac{n+1}{4}\right)^{(n+1)/2} \qquad (9.3.2.13)$$

Thus we obtain (9.3.2.1) and (9.3.2.8) and also (9.3.2.7) from (9.3.2.1) and (9.3.2.13).

Table 9.3.2.1 The Maximum Value of Optimum Weight[a]

	Maximum of Optimum Weight			Total Input Weight, W	
Number of Variables, n	Actually Obtained	Upper Bound by (9.3.2.1)	Lower Bound[b] by Theorem 9.3.1.3	Actually Obtained	Upper Bound by (9.3.2.8)
2	1	1	0.496	3	3.8
3	2	2	0.996	5	8.0
4	3	3	1.98	9	17.4
5	5	6	3.97	17	40.5
6	9	14	7.94	35	99.2
7	18	32	15.9	79	256.0
8	42	76	31.6	209	691.9
9	—	195	63.2	—	1953.1

[a] [Muroga–Toda–Kondo 62 Oc; Winder 64 Oc; Muroga–Tsuboi–Baugh 67 Ag].
[b] Since the function in Theorem 9.3.1.3 is self-dual, the value for $n - 1$ variables is used here for comparison with w_i of a function of n variables.

† Note that Δ' in (9.3.2.12) may be different from Δ of (9.3.2.5). Therefore, if $w_0, \ldots, w_n$ are to be integers, (9.3.2.5) and (9.3.2.12) must be multiplied by $\Delta\Delta'$; but the evaluation of $\Delta\Delta' w_i = \Delta_i \Delta'$ is difficult. Thus the assumption of real number weights is introduced in Theorem 9.3.2.2.

give refs to this book for # seqs

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Table 2.3.2 Number of Equivalence Classes

New #	n	0	1	2	3	4	5	6	7	8
N497	Switching functions of n or fewer variables	2	4	16	256	65,536	About 4.3×10^{9}	About 1.8×10^{19}	About 3.4×10^{38}	About 1.16×10^{77}
N45	Switching functions of exactly n variables	2	2	10	218	64,594	About 4.3×10^{9}	About 1.8×10^{19}	About 3.4×10^{38}	About 1.16×10^{77}
N492	Threshold functions of n or fewer variables	2	4	14	104	1,882	94,572	15,028,134	8,378,070,864	17,561,539,552,946
N142	Threshold functions of exactly n variables	2	2	8	72	1,536	86,080	14,487,040	8,274,797,440	17,494,930,604,032
new N484.5 = 3180	P-equivalence classes of switching functions of n or fewer variables	2	4	12	80	3,984	37,333,248	—	—	—
new N141.5 = 3181	P-equivalence classes of switching functions of exactly n variables	2	2	8	68	3,904	37,329,264	—	—	—
N308	N-equivalence classes of threshold functions of n or fewer variables	2	3	6	20	150	3,287	244,158	66,291,591	68,863,243,522
N49	N-equivalence classes of threshold functions of exactly n variables	2	1	2	9	96	2,690	226,360	64,646,855	68,339,572,672
N485	N-equivalence classes of self-dual threshold functions of $n+1$ or fewer variables	1	2	4	12	81	1,684	123,565	33,207,256	34,448,225,389
N1503	N-equivalence classes of self-dual threshold functions of exactly $n+1$ variables	1	0	1	4	46	1,322	112,519	32,267,168	34,153,652,752
N310	NP-equivalence classes of switching functions of n or fewer variables	2	3	6	22	402	1,228,158	400,507,806,843,728	—	—
N63	NP-equivalence classes of switching functions of exactly n variables	2	1	3	16	380	1,227,756	400,507,805,615,570	—	—
3182 = N979.8	NP-equivalence classes of unate functions of n or fewer variables N979.8	3	5	10	30	210	16,353	—	—	—
N272	NP-equivalence classes of threshold functions of n or fewer variables	2	3	5	10	27	119	1,113	29,375	2,730,166
N48	NP-equivalence classes of threshold functions of exactly n variables	2	1	2	5	17	92	994	28,262	2,700,791
N494	NPN-equivalence classes of switching functions of n or fewer variables	1	2	4	14	222	616,126	200,253,952,527,184	—	—
N735	NPN-equivalence classes of switching functions of exactly n variables	1	1	2	10	208	615,904	200,253,951,911,058 ?	—	—
N306.8 = 3183	NPN-equivalence classes of unate functions of n or fewer variables	2	3	6	17	112	8,282	—	—	—
N306	NPN-equivalence classes of threshold functions of n or fewer variables	1	2	3	6	15	63	567	14,755	1,366,318
N1138	NPN-equivalence classes of threshold functions of exactly n variables	1	1	1	3	9	48	504	14,188	1,351,563
N1085	Self-duality equivalence classes of switching functions of $n+1$ or fewer variables	1	1	3	7	83	109,958	—	—	—
omit	Self-duality equivalence classes of switching functions of exactly $n+1$ variables	1	0	2	4	76	109,875	—	—	—
N324	Self-duality equivalence classes of threshold functions of $n+1$ or fewer variables	1	1	2	3	7	21	135	2,470	175,428
N1417.3 = 3184	Self-duality equivalence classes of threshold functions of exactly $n+1$ variables	1	0	1	1	4	14	114	2,335	172,958

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[S. MUROGA, Threshold Logic And Its Applications, Wiley, N.Y. 1971] [AU4]

MUROGA: New Seqs & New Refs

3180
– 3187
3217
3218
2077

N1025.8 = 3185

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~~N979.1 = 3186~~

N979.5 = 3186

N979.3 = 3187

Table 7.5.1 Number of Pseudo-Threshold Functions

n, Number of Variables	1	2	3	4	5
Number of positive boolean functions of n or fewer variables	3	6	20	168	7581
Number of representative[a] positive boolean functions of n or fewer variables	3	5	10	30	210
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Weights of Threshold Functions

Type 4

# variables	1	2	3	4	5	6	7	8
W	2? [3	5	9	17	35	79	209] ns	
	1?	2	4	8	18	44		

~~3188~~
3217

Also max opt wt, ~~but too short~~

[1 2 3 5 9 18 42] ns

~~3189~~
3218

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