A7509

Scan

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4 pages

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if $4/\pi$ is reduced it is

numerator	denominator
1	1
3	2
15	13
105	76
315	263
3465	2578
45045	36979
45045	33976
765765	622637
14549535	11064338
14549535	11757173
334639305	255865444
1673196525	1346255081
5019589575	3852854518
145568097675	116752370597
4512611027925	3473755390832
4512611027925	3610501179557

R G Wilson, V

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m2006 - A7509 $4/\pi$ by $(2n+1)^2/2$

1

2

13

76

789

7734

110937

1528920

28018665

497895210

11110528485

241792844580

6361055257725

163842638377950

4964894559637425

147721447995130800

5066706567801827025

$4/\pi$ by Simple Cont. Frac.

Numerator	Denominator
4	3
5	4
9	7
14	11
219	172
452	355
32763	25732
33215	26087
331698	260515
364913	286602
6535219	5132749
6900132	5419351
20335483	15971451
27235615	21390802
156513558	122925461
183749173	144316263
340262731	267241724
3586376483	2816733503
3926639214	3083975227
11439654911	8984683957
26805949036	21053343141
547558635631	43005154677

Cont'd Fractions: 1, 3, 1, 1, 1, 15, 2, 72, 1, 9, 1, 17, 1, 2,

1, 5, 1, 1, 10, 1, 2, 2, 20, 1, 5, 1, 1, 1, 3, 3, 56, 3, 1, 2, 1, 3, 2,

3, 24, 1, 2, 9, 1, 1, 1, 2, 1, 2, 7, 1, 1, 1, 8, 2, 1, 1, 2, 13, 1, 1, 13, Fq4 7509

quicker way of obtaining the convergents (13) than to worry them out from (11). This is explained in any textbook on continued fractions.[69]

The result that Brouncker transformed into a continued fraction was the one actually given by Wallis, which was $4/\pi = \ldots$ rather than $\pi = \ldots$ as given here by (5). The continued fraction that Brouncker obtained was the pretty expression

$$4/\pi = 1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{7^2}{2 + \cfrac{9^2}{2 + \ldots}}}}} \qquad (14)$$

with convergents 1, 3/2, 15/13, 105/76, 945/789, . . .

How Brouncker obtained this result is anybody's guess; Wallis proved its equivalence with his own result (5), but his proof is so cumbersome that it almost certainly does not reflect Brouncker's derivation. Brouncker's result was later also proved by Euler (1775), whose proof amounted to the following. Consider the convergent series

$$S = a_0 + a_1 + a_1a_2 + a_1a_2a_3 + a_1a_2a_3a_4 + \ldots$$

which is easily seen to be equivalent to the continued fraction

$$S = a_0 + \cfrac{a_1}{1 - \cfrac{a_2}{1 + a_2 - \cfrac{a_3}{1 + a_3 - \ldots}}}$$

Now consider the series

$$\arctan x = x - x^3/3 + x^5/5 - x^7/7 + \ldots$$

and set $a_0 = 0$, $a_1 = x$, $a_2 = -x^2/3$, $a_3 = -3x^2/5, \ldots$; then

$$\arctan x = \cfrac{x}{1 + \cfrac{x^2}{3 - x^2 + \cfrac{9x^2}{5 - 3x^2 + \cfrac{25x^2}{7 - 5x^2 + \ldots}}}}$$

Prof. Petr Beckmann, A History of Pi π.