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Interest Rate Control Rules and Macroeconomic Stability in a Heterogeneous Two-Country Model *

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Abstract

We analyze relations between several types of interest rate control rules and equilibrium determinacy using a two-country model featuring preference and production parameters that may differ between countries, in which two kinds of goods are tradable. Such heterogeneity may violate the Taylor principle, which implies that aggressive monetary policy is desirable to attain determinate equilibrium. We evaluate the forms of interest rate control needed to attain macroeconomic stability in consideration of the heterogeneity.

Keywords and Phrases: heterogeneity, Taylor rule, open economy, equilibrium determinacy.

JEL Classification Numbers: E52, F41.

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1 Introduction

In this paper, we analyze equilibrium determinacy in a two-country model. Each country has asymmetric production technologies and preferences, and the monetary authorities may adopt different types of interest rate control rules.

Numerous studies investigate the stabilization effect of interest rate control rules utilizing small country models in open economy, as well as closed economy settings like Benhabib et. al. (2001). For example, Chang, Chen, Lai and Shaw (2008) assume an AK growth economy and a generalized Taylor rule in which the central bank controls the nominal interest rate in response to not only inflation but also the growth rate of income. [1] They show that the number of equilibrium paths is less than one, that is, equilibrium is determinate or source. [2]

Carlstrom and Fuerst (1999), Kam (2004, 2007), and Zanna (2003, 2004) examine small-open economy models that include Taylor-type monetary policy and production using only labor under conditions of sticky prices. Except for Carlstrom and Fuerst (1999), these authors clearly distinguish non-traded from tradable goods. Economies in Airaudo and Zanna (2004, 2005) [3] are also of this type, but they assume perfect competition and flexible prices. If continuous-time setting is used in their models, we only reconfirm the well-known results established in closed-economy models: the Taylor principle holds, which implies that interest rate control with an aggressive response to the inflation rate generates determinate equilibrium. They utilize discrete-time models in order to investigate how the timings of monetary dynamics [4] affect macroeconomic stability. These small-open economy models are

[1] Such a monetary policy rule is also formalized in Fujisaki and Mino (2007).

[2] However, we should note that they assume a sticky nominal interest rate.

[3] A liquidity trap in which the nominal interest rate cannot be negative is considered in Airaudo and Zanna (2004).

[4] For instance, the monetary authority controls current nominal interest rate in response to either the contemporaneous or a forward-looking inflation rate. In addition, money affecting utility is

sophisticated and yield many interesting results, but they do not clarify the effect of interaction among countries on equilibrium determinacy.

On the other hand, the role of interest rate control in a global economy model featuring two countries has also been extensively discussed in literature for examining the international economy where the policy cooperation between countries is required. In particular, many researchers, such as Leith and Wren-Levis (2009), [5] analyze New Keynesian models with sticky prices and monopolistic competition based on Clarida, Galí and Gertler (2002). They distinguish domestically produced goods from foreign-produced goods and the money in the utility is independent of consumption. Moreover, McKnight (2007a, 2007b) and McKnight and Mihailov (2007) investigate various types of two-country models by considering capital, timing of money held by household, or trade openness. However, they generally conclude that the Taylor principle tends to hold. They may have reached this conclusion as they assume parameters about preferences and production in these models to be the same in both countries.

Ono (2006) considers a two-country economy using a simple model in which two kinds of goods are tradable, money is additively separable with consumption in the utility, production is linear with regard to labor, and unemployment can emerge. He focuses on exogenous monetary policy around the steady state such that the growth rate of real money balances equals the deflation rate; that is, nominal money holdings are constant.

We revise Ono's (2006) model by using a Taylor-type policy of interest control. We specify that money holdings may not be additively-separable with consumption, prices are flexible, structural unemployment cannot occur, and production functions

the one held by household either before entering or after exiting the shop.

[5]The other examples are Batini, Levine and Pearlman (2004), Benigno and Benigno (2006), Bullard and Schalling (2006), De Fiore and Liu (2005), Ferrero et al. (2008), Gunter (2009), and Wang (2006).

involve labor and fixed productive factors. [6] In contrast to the existing two-country models, we construct a two-country model with heterogeneity. In this paper, heterogeneity pertains to the monetary authority's response to the domestic inflation rate (and other target variables such as output) via interest rate controls, and to parameters such as elasticities of labor in production and of intertemporal substitution. To simplify macroeconomic dynamics via equivalence of real interest rates between two countries, a law of one price for all goods is needed. However, this is not a natural assumption under a two-country model in which tradable and non-traded goods exist simultaneously, as in small-open economy models. If all goods are tradable as in Ono (2006) and our paper, we can naturally derive the equivalence of real interest rates. Therefore, we can easily interpret our analytical results and garner lessons for monetary policy.

We show that an appropriate cooperation of interest rate controls is required to stabilize a world economic system, primarily because controlling heterogeneity of preferences and technology is beyond a central bank's capacity. This does not necessarily mean that central banks in both countries should aggressively control nominal interest rates in response to inflation. Pursuing such policies can generate indeterminacy, because liberalization is a two-edged sword in that unstable economies may become stable and vice versa. In addition, when altering interest rates, monetary authorities should be cautious in their response to inflation, output, or depreciation in exchange rates of their domestic currencies. In particular, the effect on macroeconomic stability from differences in production between the two countries in our model depends on whether central banks use output for controlling interest rates.

[6] This follows Airaudo and Zanna (2005), but they do not claim the fixed productive factor so that profit can be non-zero. Carlstrom and Fuerst (1999) mention this shortcoming and assume that output involves labor and a fixed factor of production.

2 The Model

2.1 Households in Country 1

We assume a global economy of two countries, Country 1 and Country 2. Each produces only one kind of goods but can consume both goods by importing the other. First, we see the economic structure of Country 1 which produces Goods 1.

We denote the consumer price index (CPI) as p, the CPI-inflation rate as π, and the relative price of Goods 1 to Goods 2 $\tilde{P}$ as

$$p \equiv \left(\frac{P_1}{\alpha}\right)^{\alpha}\left(\frac{P_2}{1-\alpha}\right)^{1-\alpha}, \quad \pi = \alpha\pi_1 + (1-\alpha)\pi_2 \tag{1}$$

$$\tilde{P} = \frac{P_1}{P_2}, \tag{2}$$

where $\pi_1 \equiv \dfrac{\dot{P}_1}{P_1}$ $\left(\text{resp. } \pi_2 \equiv \dfrac{\dot{P}_2}{P_2}\right)$ is the inflation rate of the price of goods produced in Country 1 P_1 (resp. in Country 2 P_2) expressed in the domestic currency.

The production function of Goods 1 is

$$y_1 = (l_1)^{\theta_1}(\mathcal{L}_1)^{1-\theta_1}, \quad 0 < \theta_1 < 1,$$

where l_1 is the quantity of labor employed in producing Goods 1 y_1, θ_1 is the elasticity of labor employed in production and $\mathcal{L}_1$ is a fixed factor of production. This specification is based on Airaudo and Zanna (2005), who do not, however, stipulate the fixed productive factor so that profit can be non-zero. Introducing a fixed factor of production into the model, as in Carlstrom and Fuerst (1999), is beneficial in explaining situations of zero profit. In the following, we suppose that rent from the factor is distributed to households and that $\mathcal{L}_1 = 1$. Therefore, income distribution is described as follows;

$$y_1 = wl_1 + h,$$

where w is a real wage equal to the marginal production of labor $\dfrac{\theta_1 y_1}{l_1}$ and $h = (1-\theta_1)y_1$ is the rent from the fixed factor.

The household budget constraint in nominal terms is

$$\dot{B} + \dot{M} = RB + P_1(y_1 - c_1) - P_2 c_2,$$

where c_1 and c_2 are consumption of goods produced by Countries 1 and 2 respectively, R nominal interest rate, and B and M are the notations in nominal terms of b bonds and m real money balances severally. (For simplicity, we assume that lump-sum tax is zero.) We can describe this equation as

$$\frac{\dot{B} + \dot{M} - RB + T}{P_2}\frac{p}{p} = \frac{p}{P_2}(\dot{a} + \pi a - R(a - m) + \tau),$$

because

$$\frac{\dot{B} + \dot{M}}{p} = \frac{\dot{A}}{p} = \dot{a} + \pi a.$$

Using

$$\frac{p}{P_2} = \frac{\tilde{P}^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$$

from (1) and (2), we obtain

$$\dot{a} = (R - \pi)a - Rm + \alpha^{\alpha}(1-\alpha)^{1-\alpha}\tilde{P}^{-\alpha}[\tilde{P}(y_1 - c_1) - c_2], \tag{3}$$

where $a \equiv b + m$ denotes real financial assets.

The optimization problem of a representative household in Country 1 is

$$\max \quad \int_0^{\infty} u(c, m, l_1)e^{-\rho t}dt, \quad \rho > 0,$$

subject to (3), where the instantaneous utility is

$$u(c, m, l_1) = \frac{(c^{\gamma}m^{1-\gamma})^{1-\sigma}}{1-\sigma} + \psi(1 - l_1), \quad 0 < \gamma < 1, \quad \sigma > 0, \quad \psi > 0,$$

$(1 - l_1)$ leisure, ρ the time discount rate, σ the inverse of intertemporal elasticity of substitution (IES), and c is the consumption aggregator given by

$$c = (c_1)^{\alpha}(c_2)^{1-\alpha}, \quad 0 < \alpha < 1, \tag{4}$$

The Hamiltonian function is

$$\mathcal{H} = \frac{(c^{\gamma}m^{1-\gamma})^{1-\sigma}}{1-\sigma} + \psi(1-l_1) + \lambda\{(R-\pi)a - Rm + \alpha^{\alpha}(1-\alpha)^{1-\alpha}\tilde{P}^{-\alpha}[\tilde{P}(y_1-c_1)-c_2]\},$$

where λ denotes the shadow value of assets. The first-order conditions are

$$\gamma\alpha\frac{(c^{\gamma}m^{1-\gamma})^{1-\sigma}}{c_1} = \alpha^{\alpha}(1-\alpha)^{1-\alpha}\tilde{P}^{1-\alpha}\lambda, \tag{5}$$

$$\gamma(1-\alpha)\frac{(c^{\gamma}m^{1-\gamma})^{1-\sigma}}{c_2} = \alpha^{\alpha}(1-\alpha)^{1-\alpha}\tilde{P}^{-\alpha}\lambda, \tag{6}$$

$$(1-\gamma)\frac{(c^{\gamma}m^{1-\gamma})^{1-\sigma}}{m} = \lambda R, \tag{7}$$

$$\alpha^{\alpha}(1-\alpha)^{1-\alpha}\tilde{P}^{1-\alpha}\lambda\theta_1(l_1)^{-(1-\theta_1)} = \psi, \tag{8}$$

$$\dot{\lambda} = [\rho + \pi - R]\lambda, \tag{9}$$

together with the transversality condition, $\lim_{t\to\infty} e^{-\rho t}\lambda_t a_t = 0$. These equations severally show the equivalence of marginal benefits and costs for consumption goods, money, labor, and asset holdings. We can rewrite them as follows:

$$\frac{c_2}{c_1} = \frac{1-\alpha}{\alpha}\tilde{P}, \tag{10}$$

$$m = \frac{1-\gamma}{\gamma}\frac{c}{R}, \tag{11}$$

$$\gamma\alpha\frac{(c^{\gamma}m^{1-\gamma})^{1-\sigma}}{c_1} = \frac{\psi}{\theta_1(l_1)^{-(1-\theta_1)}}. \tag{12}$$

These respectively imply the optimal conditions for the constant ratio of nominal consumption expenditures, optimal demand for real money holdings, and the marginal rate of substitution between consumption and leisure as the residual of labor for producing Goods 1.

2.2 Households in Country 2

The economic structure of Country 2, which produces Goods 2, is the same as that of Country 1. Therefore, when parameters ρ, α, γ and ψ are the same as in Country 1, the Hamiltonian of Country 2's household maximization problem is

$$\mathcal{H}^* = \frac{((c^*)^\gamma (m^*)^{1-\gamma})^{1-\sigma^*}}{1-\sigma^*} + \psi(1-l_2^*) + \lambda^*\{(R^*-\pi^*)a^* - R^*m^* + \alpha^\alpha(1-\alpha)^{1-\alpha}\tilde{P^*}^{-\alpha}[-\tilde{P^*}c_1^* + (y_2^*-c_2^*)]\}.$$

We represent variables and parameters for Country 2 with asterisks. For example, the consumption index is described as $c^* = (c_1^*)^\alpha (c_2^*)^{1-\alpha}$. The method of translating the nominal terms of the budget constraint into real terms is similar to that for Country 1. Variables for the price level in Country 2 are

$$p^* \equiv \left(\frac{P_1^*}{\alpha}\right)^\alpha \left(\frac{P_2^*}{1-\alpha}\right)^{1-\alpha} = \left(\frac{\varepsilon P_1}{\alpha}\right)^\alpha \left(\frac{\varepsilon P_2}{1-\alpha}\right)^{1-\alpha} = \varepsilon p, \quad \tilde{P^*} = \frac{\varepsilon P_2}{\varepsilon P_1} = \tilde{P}, \quad (13)$$

where ε is the nominal exchange rate. These equations imply that the law of one price holds. For example, the price of Goods 1 is transcribed as P_1 yen in Japan and $P_1^* = \dfrac{P_1}{\varepsilon}$ dollars in the United States. This is an acceptable assumption because both goods are tradable.

Goods 2 is produced by technology such that

$$y_2^* = (l_2^*)^{\theta_2}(\mathcal{L}_2^*)^{1-\theta_2}, \quad 0 < \theta_2 < 1,$$

where θ_2^* is the elasticity of labor employed in production and $\mathcal{L}_2^*$ is a fixed factor of production for Goods 2. Again, we assume that rent from the factor is distributed to households and that $\mathcal{L}_2^* = 1$ for simplicity. Then, income distribution is described as follows;

$$y_2^* = w^* l_2^* + h^*,$$

where w^* is a real wage equal to the marginal production of labor $\dfrac{\theta_2^* y_2^*}{l_2^*}$ and h^* is the rent from the fixed factor.

Denoting λ^* as the shadow value of assets in Country 2, we can write the first-order conditions as follows:

$$\gamma\alpha\frac{((c^*)^\gamma(m^*)^{1-\gamma})^{1-\sigma^*}}{c_1^*} = \alpha^\alpha(1-\alpha)^{1-\alpha}\tilde{P}^{1-\alpha}\lambda^*, \tag{14}$$

$$\gamma(1-\alpha)\frac{((c^*)^\gamma(m^*)^{1-\gamma})^{1-\sigma^*}}{c_2^*} = \alpha^\alpha(1-\alpha)^{1-\alpha}\tilde{P}^{-\alpha}\lambda^*, \tag{15}$$

$$(1-\gamma)\frac{((c^*)^\gamma(m^*)^{1-\gamma})^{1-\sigma^*}}{m^*} = \lambda^* R^*, \tag{16}$$

$$\alpha^\alpha(1-\alpha)^{1-\alpha}\tilde{P}^{-\alpha}\lambda^*\theta_2^*(l_2^*)^{-(1-\theta_2^*)} = \psi, \tag{17}$$

$$\dot{\lambda^*} = [\rho + \pi^* - R^*]\lambda^*. \tag{18}$$

These can be rewritten such that

$$\frac{c_2^*}{c_1^*} = \frac{1-\alpha}{\alpha}\tilde{P}, \tag{19}$$

$$m^* = \frac{1-\gamma}{\gamma}\frac{c^*}{R^*}, \tag{20}$$

$$\gamma(1-\alpha)\frac{((c^*)^\gamma(m^*)^{1-\gamma})^{1-\sigma^*}}{c_2^*} = \frac{\psi}{\theta_2^*(l_2^*)^{-(1-\theta_2^*)}}. \tag{21}$$

These equations are similar to (5)−(12), except that labor is used in the production of Goods 2, and the transversality condition is $\lim_{t\to\infty} e^{-\rho t}\lambda_t^* a_t^* = 0$. .

2.3 Monetary Policy and Interest-Rate Conditions

As in Taylor (1993), we suppose the central bank in each country controls the nominal interest rate in response to its aggregate domestic rate of inflation;

$$R = R(\pi) = \eta_\pi(\pi - \bar{\pi}) + \bar{R}, \quad \eta_\pi \geq 0, \tag{22}$$

$$R^* = R^*(\pi^*) = \eta_\pi^*(\pi^* - \bar{\pi}^*) + \bar{R}^*, \quad \eta_\pi^* \geq 0, \tag{23}$$

where $\bar{\pi}$ and $\bar{\pi}^*$ are the non-negative target rates of inflation in Countries 1 and 2, respectively, and $\bar{R} = \bar{\pi} + \rho$ and $\bar{R}^* = \bar{\pi}^* + \rho$ around the steady state which implies

that $\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\lambda^*}}{\lambda^*} = 0$ as confirmed from Equations (9) and (18). We rewrite (22) and (23) in the following manner:

$$\pi = \pi(R), \quad \pi'(R) = \frac{1}{\eta_\pi},$$

$$\pi^* = \pi^*(R^*), \quad \pi^{*'}(R^*) = \frac{1}{\eta_\pi^*}.$$

Under this formulation, we define active (resp. passive) monetary policy as $\eta_\pi > 1$ or $\eta_\pi^* > 1$ (resp. $\eta_\pi < 1$ or $\eta_\pi^* < 1$), which indicates that the real interest rate is an increasing (resp. a decreasing) function of inflation and thus of the nominal interest rate. Heterogeneity can be assumed in the monetary authority's adjustment of the interest rate in response to inflation as well as parameters of preferences and production. That is, we allow $\eta_\pi \neq \eta_\pi^*$.

The interest-parity condition is

$$R = \epsilon + R^*, \tag{24}$$

where $\epsilon \equiv \frac{\dot{\varepsilon}}{\varepsilon}$ is the rate of devaluation in the nominal exchange rate ε. From the law of one price (13),

$$\pi = \epsilon + \pi^*, \tag{25}$$

and thus we obtain a non-arbitrage condition

$$r = R - \pi = R^* - \pi^*, \tag{26}$$

where r denotes the real interest rate common to both countries. [7] Therefore,

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\lambda^*}}{\lambda^*} = \rho + \pi - R = \rho - r \tag{27}$$

holds from (9) and (18). This means that the ratio between shadow values λ and λ^* is a positive constant $\Phi > 0$ determined by initial assets a_0 and a_0^*, $\lambda = \Phi\lambda^*$.

[7] This condition is revised if there are non-traded goods. In such a case, the law of one price is not generally plausible.

We find that the nominal interest rate in Country 2 R^*, the currency devaluation rate ϵ, and the real rate of interest r are functions of R:

$$R^* = R^*(R), \quad R^{*'}(R) = \frac{\eta_\pi^*(\eta_\pi - 1)}{\eta_\pi(\eta_\pi^* - 1)},$$

$$r = r(R), \quad r'(R) = \frac{\eta_\pi - 1}{\eta_\pi}.$$

$$\epsilon = \epsilon(R), \quad \epsilon'(R) = \frac{\eta_\pi^* - \eta_\pi}{\eta_\pi(\eta_\pi^* - 1)},$$

For example, if both central banks adopt passive monetary policy and the response to inflation is stronger in Country 2 (that is, $\eta_\pi < \eta_\pi^* < 1$), then

$$R^{*'}(R) > 0, \quad \epsilon'(R) < 0 \quad \text{and} \quad r'(R) < 0.$$

When inflation in Country 1 decreases, nominal interest rates in both countries fall and their common real rate of interest should rise in order to satisfy the non-arbitrage condition (26). The extent of decline in the nominal interest rate as the opportunity cost for holding money is comparatively larger in Country 2. This leads to the depreciation of Country 1's currency (that is, ϵ increases), according to the interest-parity condition (24) or the law of one price (25). In contrast, $R^{*'}(R) = 1$ and $\epsilon'(R) = 0$ holds under $\eta_\pi = \eta_\pi^*$ since exchange rate fluctuation via heterogeneity of monetary policy does not occur.

3 Equilibrium Determinacy

3.1 Equilibrium

We see the effect of heterogeneity on macroeconomic stability by allowing that $\sigma \neq \sigma^*$ and $\theta_1 \neq \theta_2^*$. From Equations (5)−(21), important variables can be described as functions of R and λ in Country 1 (R^* and λ^* in Country 2):

$$c = \mathcal{C}(R^{(1-\gamma)(1-\sigma)}\lambda)^{-\frac{1}{\sigma}}, \tag{28}$$

$$m = \frac{1-\gamma}{\gamma}\mathcal{C}R^{-\frac{1-\gamma+\gamma\sigma}{\sigma}}\lambda^{-\frac{1}{\sigma}}, \tag{29}$$

$$c^* = \mathcal{C}^*((R^*)^{(1-\gamma)(1-\sigma^*)}\lambda^*)^{-\frac{1}{\sigma^*}}, \tag{30}$$

$$m^* = \frac{1-\gamma}{\gamma}\mathcal{C}^*(R^*)^{-\frac{1-\gamma+\gamma\sigma^*}{\sigma^*}}(\lambda^*)^{-\frac{1}{\sigma}}, \tag{31}$$

where

$$\mathcal{C} \equiv [\gamma^{1-(1-\gamma)(1-\sigma)}(1-\gamma)^{(1-\gamma)(1-\sigma)}]^{\frac{1}{\sigma}} \text{ and } \mathcal{C}^* \equiv [\gamma^{1-(1-\gamma)(1-\sigma^*)}(1-\gamma)^{(1-\gamma)(1-\sigma^*)}]^{\frac{1}{\sigma^*}}.$$

Respective market equilibria of Goods 1 and 2 are

$$y_1 = c_1 + c_1^*, \quad y_2^* = c_2 + c_2^*. \tag{32}$$

Using (10), (19), and (32), we derive

$$\frac{y_2^*}{y_1} = \frac{c_2^*}{c_1^*} = \frac{c_2}{c_1} = \frac{1-\alpha}{\alpha}\tilde{P}. \tag{33}$$

Then, from (4) or $c^* = (c_1^*)^{\alpha}(c_2^*)^{1-\alpha}$ and (32),

$$c_1 = c\left(\frac{y_1}{y_2^*}\right)^{1-\alpha}, \quad c_1^* = c^*\left(\frac{y_1}{y_2^*}\right)^{1-\alpha}. \tag{34}$$

$$c_2 = c\left(\frac{y_1}{y_2^*}\right)^{-\alpha}, \quad c_2^* = c^*\left(\frac{y_1}{y_2^*}\right)^{-\alpha}. \tag{35}$$

Combining (32) with (34) or (35), we obtain

$$(y_1)^{\alpha}(y_2^*)^{1-\alpha} = c + c^*. \tag{36}$$

The sum of consumption indices in the two countries is equal to the "production index", which is similar to the consumption index. In addition, from (7), (16), and (33),

$$\frac{c^*}{c} = \frac{c_1^*}{c_1} = \frac{c_2^*}{c_2}, \tag{37}$$

$$\frac{(c^{\gamma}m^{1-\gamma})^{1-\sigma}}{((c^*)^{\gamma}(m^*)^{1-\gamma})^{1-\sigma^*}}\frac{m^*}{m} = \frac{\lambda R}{\lambda^* R^*}, \tag{38}$$

$$\tilde{P}\frac{\lambda}{\lambda^*} = \frac{\theta_2^*(l_2^*)^{-(1-\theta_2^*)}}{\theta_1(l_1)^{-(1-\theta_1)}} = \frac{\alpha}{1-\alpha}\frac{(c^{\gamma}m^{1-\gamma})^{1-\sigma}}{((c^*)^{\gamma}(m^*)^{1-\gamma})^{1-\sigma^*}}\frac{c_2^*}{c_1}. \tag{39}$$

From (33) and (39), we acquire

$$l_1 = \frac{\alpha}{1-\alpha}\frac{\theta_1}{\theta_2^*}\frac{\lambda}{\lambda^*}l_2^*, \tag{40}$$

and thus

$$\tilde{P} = \left(\frac{\alpha}{1-\alpha}\right)^{1-\theta_1}\left(\frac{\theta_2^*}{\theta_1}\right)^{\theta_1}\left(\frac{\lambda}{\lambda^*}\right)^{-\theta_1}(l_2^*)^{\theta_2^*-\theta_1}. \tag{41}$$

Then, respective labor supplies in the two countries are functions of λ and λ^*:

$$l_1 = \frac{\alpha}{1-\alpha}\frac{\theta_1}{\theta_2^*}[\Gamma\lambda^{1-(1-\alpha)\theta_2^*}(\lambda^*)^{(1-\alpha)\theta_2^*}]^{\frac{1}{1-\theta_2^*+\alpha(\theta_2^*-\theta_1)}}, \tag{42}$$

$$l_2^* = [\Gamma\lambda^{\alpha\theta_1}(\lambda^*)^{1-\alpha\theta_1}]^{\frac{1}{1-\theta_2^*+\alpha(\theta_2^*-\theta_1)}}, \tag{43}$$

where

$$\Gamma \equiv \frac{\alpha^\alpha(1-\alpha)^{1-\alpha}\theta_2^*}{\psi}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha(1-\theta_1)}\left(\frac{\theta_1}{\theta_2^*}\right)^{\alpha\theta_1}.$$

Therefore, the relative price function becomes

$$\tilde{P} = \frac{P_1}{P_2} = \left(\frac{\alpha}{1-\alpha}\right)^{1-\theta_1}\left(\frac{\theta_2^*}{\theta_1}\right)^{\theta_1}\Gamma^{\frac{\theta_2^*-\theta_1}{1-\theta_2^*+\alpha(\theta_2^*-\theta_1)}}\lambda^{-\frac{\theta_1(1-\theta_2^*)}{1-\theta_2^*+\alpha(\theta_2^*-\theta_1)}}(\lambda^*)^{\frac{\theta_2^*(1-\theta_1)}{1-\theta_2^*+\alpha(\theta_2^*-\theta_1)}},$$

and then

$$\frac{\dot{\tilde{P}}}{\tilde{P}} = \frac{\theta_2^*-\theta_1}{1-\theta_2^*+\alpha(\theta_2^*-\theta_1)}\frac{\dot{\lambda}}{\lambda} = \frac{\theta_1-\theta_2^*}{1-\theta_2^*+\alpha(\theta_2^*-\theta_1)}(r-\rho)$$

from (27). Note that

$$1-\theta_2^*+\alpha(\theta_2^*-\theta_1) = \alpha(1-\theta_1)+(1-\alpha)(1-\theta_2) \in (0,1),$$

because $0 < \theta_1 < 1$ and $0 < \theta_2 < 1$. A higher real interest rate implies a lower shadow value of assets, because higher r indicates a larger gain per asset and thus the value of one unit of asset declines. This occurs because the need to generate more foreign demand in order to accumulate assets subsides and production in both countries therefore decreases. From (27), (42), and (43),

$$\frac{\dot{l_1}}{l_1} = \frac{\dot{l_2^*}}{l_2^*} = \frac{1}{1-\theta_2^*+\alpha(\theta_2^*-\theta_1)}\frac{\dot{\lambda}}{\lambda}$$

holds. That is, the growth rate of labor as a productive factor is equal between the two countries, since marginal utilities of leisure equal the marginal productivities of labor for the optimality shown in (8) and (17) and are constant in both countries. When $\theta_1 < \theta_2^*$, the extent of decrease in supply of Goods 2 with diminishing labor is larger, and then the relative price of Goods 1 decreases since the ratio between nominal output in the two countries is constant from (33).

The equilibrium condition for bond market is

$$b + b^* = 0,$$

since bonds are IOUs between the home and foreign households. Then, combining this condition, households' budget constraint, and equilibrium of goods market, we acquire the equilibrium condition for money which holds due to the Walras' law:

$$\dot{m} + \dot{m}^* = -\pi m - \pi^* m^*.$$

3.2 System Equation

As in (28)−(31), consumption indices c and c^* are functions of the nominal interest rate and shadow values in their respective countries. However, we have already shown that $\lambda = \Phi\lambda^*$ and $R^* = R^*(R)$. From equation (36), the dynamic system equation is ultimately a function of one jump variable R,

$$\dot{R} = -\frac{\lambda(R)}{\lambda'(R)}[R - \pi(R) - \rho], \tag{44}$$

where

$$\lambda'(R) = \frac{d\lambda}{dR} = -\frac{c\dfrac{(1-\gamma)(1-\sigma)}{\sigma} + c^*\dfrac{(1-\gamma)(1-\sigma^*)}{\sigma^*}\dfrac{R^{*\prime}(R)R}{R^*(R)}}{\dfrac{c}{\sigma} + \dfrac{c^*}{\sigma^*} + (c+c^*)\left[\dfrac{1}{1-\theta_2^*+\alpha(\theta_2^*-\theta_1)} - 1\right]}\frac{\lambda}{R},$$

because differentiating equation (36), we obtain

$$(y_1)^{\alpha}(y_2^*)^{1-\alpha}\left[\frac{1}{1-\theta_2^*+\alpha(\theta_2^*-\theta_1)}-1\right]\frac{\dot{\lambda}}{\lambda}= \\ -\frac{(1-\gamma)(1-\sigma)}{\sigma}c\frac{\dot{R}}{R}-\frac{c}{\sigma}\frac{\dot{\lambda}}{\lambda}-\frac{(1-\gamma)(1-\sigma^*)}{\sigma^*}c^*\frac{\dot{R}^*}{R^*}-\frac{c^*}{\sigma^*}\frac{\dot{\lambda}^*}{\lambda^*}. \quad (45)$$

We evaluate Equation (44) around the steady state in order to examine local determinacy:

$$\dot{R}_R|_{ss}=-\frac{\lambda(\bar{R})}{\lambda'(\bar{R})}[1-\pi'(\bar{R})],$$

where variables with an upper bar denote the steady-state values given the target rates of inflation, $\bar{\pi}$ and $\bar{\pi}^*$. Equilibrium determinacy where the equilibrium path is unique is realized if $\dot{R}_R|_{ss}>0$. It implies a stable economy in that non-fundamental expectations cannot affect economic fluctuations. Otherwise, equilibrium is indeterminate as multiple equilibrium paths emerge. We conclude that

$$\text{sign}[\dot{R}_R|_{ss}]=\text{sign}\left[\frac{\dfrac{\eta_\pi-1}{\eta_\pi}}{c(\bar{R})\dfrac{(1-\gamma)(1-\sigma)}{\sigma}+c^*(\bar{R})\dfrac{(1-\gamma)(1-\sigma^*)}{\sigma^*}\dfrac{\bar{R}}{R^*(\bar{R})}\dfrac{\eta_\pi^*(\eta_\pi-1)}{\eta_\pi(\eta_\pi^*-1)}}\right],$$

and thus we roughly summarize the results of equilibrium determinacy in the following propositions and Table 1:

Proposition 1 *If the value of IES is 1 in either country, the other country's central bank can make equilibrium determinate by invoking a policy rule such that* $\dfrac{\eta_\pi-1}{1-\sigma}>0$ $\left(\text{ or } \dfrac{\eta_\pi^*-1}{1-\sigma^*}>0\right)$.

Proposition 2 *When* $\sigma<1$ *and* $\sigma^*<1$ *(resp.* $\sigma>1$ *and* $\sigma^*>1$*) holds, both countries should adopt an active (resp. passive) policy in order to assure determinate equilibrium.*

Proposition 3 *In situations where* $(1-\sigma)(1-\sigma^*)<0$*, determinate equilibrium is generated by a combination of passive interest-rate control in the country pre-*

senting an IES value below 1 and aggressive monetary policy in the other country. Indeterminacy inevitably emerges under the converse combination of policies.

When $\sigma = \sigma^* = 1$, that is, consumption and money are additively separable in both countries, consumption indices are decreasing functions of only the shadow value of assets from (28) and (30). Therefore, the value is uniquely determined by equilibrium in the goods market even if it does not approach the steady state:

Proposition 4 *If $\sigma = \sigma^* = 1$, equilibrium is necessarily determinate regardless of monetary policy.*

3.3 Implications

Propositions 1 and 4 are similar to the results under the one-country model or the economy with endowment in Benhabib et al. (2001). In addition, from Propositions 2 and 3, we find that economic liberalization can overcome indeterminacy in one country, but a stable economy may owe the risk of instability by the trade liberalization. Such various conclusions via preference do not appear in the literature involving two-country models, which often assume that preferences between consumption and money are additively separable.

Let us consider these results intuitively. Suppose that $\sigma < 1$ and $\sigma^* > 1$ (i.e., consumption and money are complements in Country 1 and substitutes in Country 2), and that the inflation rate in Country 1 subsides.

If only Country 1 adopts passive monetary policy, its real interest rate rises. The productions relative to consumption for asset accumulation falls as in the previous subsection, but consumption in both countries increases with higher inflation in Country 2. Therefore, Equation (36) does not hold, which implies that equilibrium is indeterminate.

When both central banks adopt active monetary policy, the real and nominal

interest rates decline in both countries. Consumption increases in Country 1, while decreases in Country 2. In addition, production becomes larger relative to consumption in the two countries. Therefore, whether Equation (36) is satisfied depends on the heterogeneity of not only preference parameters but also monetary policy. Determinacy tends to hold if the response of interest-rate control in Country 1 η_π is higher, because a large increase in consumption in Country 1 through monetary policy is required for equilibrium in the goods-market. This is similar under passive interest rate control in both monetary authorities, but η_π should be weaker in order to restrain the effect of decreasing consumption in Country 1. Since preference parameters cannot be controlled by monetary authorities and inequality between σ and σ^* may emerge in general, naive adjustment of interest rate control is needed for a stable economy in cases where either determinacy or indeterminacy may emerge.

To check the robustness of the results, for example, in which heterogeneity of production does not affect the results in this section, we formulate other types of a Taylor rule in the next section.

4 Other Types of Interest Rate Controls

4.1 Response to Depreciation

Because currency depreciation has the same effect as higher inflation under the law of one price, we consider a monetary policy rule such that nominal interest rates also respond to the depreciation rate ϵ as in Ball (1998):

$$R = R(\pi) = \eta_\pi(\pi - \bar{\pi}) + \eta_\epsilon(\epsilon - \bar{\epsilon}) + \bar{R}, \quad \eta_\pi \geq 0, \quad \eta_\epsilon \geq 0, \tag{46}$$

$$R^* = R^*(\pi^*) = \eta^*_\pi(\pi^* - \bar{\pi}^*) - \eta^*_\epsilon(\epsilon - \bar{\epsilon}) + \bar{R}^*, \quad \eta^*_\pi \geq 0, \quad \eta^*_\epsilon \geq 0. \tag{47}$$

This type of monetary policy using the exchange rate is peculiar to an open economy. Utilizing this method, we can stabilize a multi-country global economy, in which each

is unstable under the closed economy if the monetary authority uses only inflation as its meridian for setting policy.

From this formulation, the variables as functions of R can be described in the followings:

$$\pi = \pi(R), \quad \pi'(R) = \frac{\eta_\pi^* - 1 + \eta_\epsilon^* + \eta_\epsilon}{\eta_\pi(\eta_\pi^* - 1) + \eta_\pi\eta_\epsilon^* + \eta_\pi^*\eta_\epsilon},$$
$$\epsilon = \epsilon(R), \quad \epsilon'(R) = \frac{\eta_\pi^* - \eta_\pi}{\eta_\pi(\eta_\pi^* - 1) + \eta_\pi\eta_\epsilon^* + \eta_\pi^*\eta_\epsilon}.$$
$$R^* = R^*(R), \quad R^{*'}(R) = \frac{\eta_\pi^*(\eta_\pi - 1) + \eta_\pi\eta_\epsilon^* + \eta_\pi^*\eta_\epsilon}{\eta_\pi(\eta_\pi^* - 1) + \eta_\pi\eta_\epsilon^* + \eta_\pi^*\eta_\epsilon},$$
$$r'(R) = 1 - \pi'(R) = \frac{(\eta_\pi^* - 1)(\eta_\pi - 1) + \eta_\epsilon^*(\eta_\pi - 1) + \eta_\epsilon(\eta_\pi^* - 1)}{\eta_\pi(\eta_\pi^* - 1) + \eta_\pi\eta_\epsilon^* + \eta_\pi^*\eta_\epsilon}.$$

Under $\eta_\pi = \eta_\pi^*$, the following still holds even if both η_ϵ and η_ϵ^* are positive:

$$\pi'(R) = \frac{1}{\eta_\pi}, \quad R^{*'}(R) = 1, \quad \epsilon'(R) = 0, \quad \text{and} \quad r'(R) = \frac{\eta_\pi - 1}{\eta_\pi}.$$

That is, heterogeneity of monetary policy as a source of fluctuation in currency's value is a condition for which the responses to the depreciation rate, η_ϵ and η_ϵ^*, have a significant effect. If inflation rates are lower under passive interest rate control in both countries and $\eta_\pi < \eta_\pi^* < 1$, a decrease in the nominal interest rate in Country 2 is larger so that Country 1's currency is depreciated as described in Section 2.3, making the nominal rate in Country 1 higher if $\eta_\epsilon > 0$. Then, R and R^* can move in opposite directions, that is, when η_ϵ is sufficiently strong, $R^{*'}(R) < 0$ may hold even though $\eta_\pi < \eta_\pi^* < 1$.

In addition, the real interest rate surely increases with the nominal rate only if both countries adopt active monetary policy. Otherwise, the sign of $r'(R)$ is ambiguous, and thus the relation between economic stability and monetary policy may be more complicated, although the reduced form of the system equation is apparently the same as in (44):

Proposition 5 *Under assertive interest rate controls with positive response to the depreciation rate of currency in two countries, equilibrium is determinate when* $\sigma <$

1 *and* $\sigma^* < 1$*, while it is indeterminate if both* σ *and* σ^* *exceed 1. Otherwise, equilibrium can be determinate or indeterminate.*

4.2 Generalized Taylor Rule

Next, we investigate the effect on equilibrium determinacy from Taylor rules such that central banks respond to both inflation and production in each country:

$$R = R(\pi) = \eta_\pi(\pi - \bar{\pi}) + \eta_y(y_1 - \bar{y}_1) + \bar{R}, \quad \eta_\pi \geq 0, \quad \eta_y \geq 0, \tag{48}$$

$$R^* = R^*(\pi^*) = \eta^*_\pi(\pi^* - \bar{\pi}^*) + \eta^*_y(y^*_2 - \bar{y}^*_2) + \bar{R}^*, \quad \eta^*_\pi \geq 0, \quad \eta^*_y \geq 0. \tag{49}$$

This is an original style suggested in Taylor (1993). Note that $\bar{y}_1$ and $\bar{y}^*_2$ are the steady-state values of output, not the levels of the natural rate. From the non-arbitrage condition (26) and the reduced forms of labor (42) and (43), we obtain

$$R^*(R, \lambda, \lambda^*) = \frac{\eta^*_\pi}{\eta^*_\pi - 1}\left[\frac{\eta_\pi - 1}{\eta_\pi}(R - \bar{R}) + \frac{\eta_y}{\eta_\pi}(\{l_1(\lambda, \lambda^*)\}^{\theta_1} - \bar{y}_1) \right.$$
$$\left. - \frac{\eta^*_y}{\eta^*_\pi}(\{l^*_2(\lambda, \lambda^*)\}^{\theta^*_2} - \bar{y}^*_2) + \bar{R}^*\right],$$

and then

$$\frac{\dot{R}^*}{R^*} = \frac{\eta^*_\pi}{\eta^*_\pi - 1}\left[\frac{\eta_\pi - 1}{\eta_\pi}\frac{R}{R^*}\frac{\dot{R}}{R} + \frac{1}{1 - \theta^*_2 + \alpha(\theta^*_2 - \theta_1)}\frac{1}{R^*}\left(\theta_1\frac{\eta_y}{\eta_\pi}y_1 - \theta^*_2\frac{\eta^*_y}{\eta^*_\pi}y^*_2\right)\frac{\dot{\lambda}}{\lambda}\right]. \tag{50}$$

Combining (45) and (50), we obtain the system equation consisted by one jump variable R as in the previous sections. The external form is the same as in Equation (44), but the relation between λ and R is not. Concretely,

$$\lambda'(R) = \frac{d\lambda}{dR} = -\frac{c\dfrac{(1-\gamma)(1-\sigma)}{\sigma} + c^*\dfrac{(1-\gamma)(1-\sigma^*)}{\sigma^*}\dfrac{\eta^*_\pi(\eta_\pi - 1)}{\eta_\pi(\eta^*_\pi - 1)}\dfrac{R}{R^*(R)}}{F(R, R^*, \lambda, \lambda^*)}\frac{\lambda}{R},$$

and

$$r = R - \pi(R, \lambda, \lambda^*) = \frac{\eta_\pi - 1}{\eta_\pi}(R - \bar{R}) + \frac{\eta_y}{\eta_\pi}(\{l_1(\lambda, \lambda^*)\}^{\theta_1} - \bar{y}_1) + \rho,$$

hold, where

$$F(R,R^*,\lambda,\lambda^*) \equiv \frac{c}{\sigma}+\frac{c^*}{\sigma^*}+(c+c^*)\left[\frac{1}{1-\theta_2^*+\alpha(\theta_2^*-\theta_1)}-1\right]+ \\ \frac{\eta_\pi^*}{\eta_\pi^*-1}\frac{(1-\gamma)(1-\sigma^*)}{\sigma^*}\frac{c^*}{R^*}\cdot\frac{1}{1-\theta_2^*+\alpha(\theta_2^*-\theta_1)}\left(\theta_1\frac{\eta_y}{\eta_\pi}y_1-\theta_2^*\frac{\eta_y^*}{\eta_\pi^*}y_2^*\right),$$

and

$$r'(R)=\frac{dr}{dR}=\left(\frac{\eta_\pi-1}{\eta_\pi}+\frac{\eta_y}{\eta_\pi}\frac{\theta_1}{1-\theta_2^*+\alpha(\theta_2^*-\theta_1)}\frac{y_1}{\lambda(R)}\lambda'(R)\right).$$

The approximation of the system equation around the steady state is

$$\begin{aligned}\dot{R}_R|_{ss} &= \frac{r'(\bar{R})\cdot\lambda(\bar{R})}{-\lambda'(\bar{R})} \\ &= \left[\frac{\eta_\pi-1}{\eta_\pi}\frac{1}{-\lambda'(\bar{R})}-\frac{\eta_y}{\eta_\pi}\frac{\theta_1}{1-\theta_2^*+\alpha(\theta_2^*-\theta_1)}\frac{\bar{y_1}}{\lambda(\bar{R})}\right]\lambda(\bar{R}),\end{aligned}$$

and the results are summarized in the following propositions and Tables 2 and 3:

Proposition 6 *If* $\left(\theta_1\frac{\eta_y}{\eta_\pi}y_1-\theta_2^*\frac{\eta_y^*}{\eta_\pi^*}y_2^*\right)>0$, *indeterminacy emerges when both* $\sigma^*=1$ *and* $\frac{\eta_\pi-1}{1-\sigma}<0$ *are satisfied. Othewise, equilibrium can be either determinate or indeterminate.*

Proposition 7 *When* $\left(\theta_1\frac{\eta_y}{\eta_\pi}y_1-\theta_2^*\frac{\eta_y^*}{\eta_\pi^*}y_2^*\right)\leq 0$, $\eta_y>0$, *and* $\eta_y^*>0$, *determinacy under the zero-response of monetary policy to income may be violated.*

To interpret the result, we remember the situation used in Section 3.3. In the case of $\left(\theta_1\frac{\eta_y}{\eta_\pi}y_1-\theta_2^*\frac{\eta_y^*}{\eta_\pi^*}y_2^*\right)\leq 0$ such as $\eta_y^*>\eta_y=0$, the real rate of interest in Country 2 totally should be higher even if the income decreases so that the essential mechanism of indeterminacy does not change. On the other hand, the common real interest rates and thus the nominal rate of interest and consumption in Country 2 may diminish according to smaller income, when $\left(\theta_1\frac{\eta_y}{\eta_\pi}y_1-\theta_2^*\frac{\eta_y^*}{\eta_\pi^*}y_2^*\right)>0$ such as $\eta_y>\eta_y^*=0$, which can make equilibrium determinate easier if η_y is large enough.

5 Conclusion

We analyze equilibrium determinacy in a two-country model with heterogeneity in interest rate control rules, production technologies, and preferences.

This paper shows that monetary policy plays a more important role for stabilizing economy when heterogeneity exists, because authorities cannot control differences in preferences and technology. We should note that active interest rate adjustments can generate indeterminate equilibrium, and central banks should be cautious about the degree of response to inflation, output, or depreciation rate. Especially, results suggest that monetary stabilization policies can be designed by utilizing heterogeneity of preference and production.

The findings in this paper suggest several themes for future research, including the existence of non-traded goods, preference formulations such as an endogenous time discount rate, habit persistence, or socia-status as in Farmer and Lahiri (2005), and discrete-time analysis as in Airaudo and Zanna (2005).

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Table 1: Equilibrium determinacy under the standard case in Section 3

$\sigma^* = 1$	$\sigma < 1$	$\sigma > 1$
$\eta_\pi > 1$	D	I
$\eta_\pi < 1$	I	D

$\sigma = 1$	$\sigma^* < 1$	$\sigma^* > 1$
$\eta^*_\pi > 1$	D	I
$\eta^*_\pi < 1$	I	D

$\sigma < 1, \sigma^* < 1$	$\eta^*_\pi < 1$	$\eta^*_\pi > 1$
$\eta_\pi > 1$	D, I	D
$\eta_\pi < 1$	I	D, I

$\sigma < 1, \sigma^* > 1$	$\eta^*_\pi < 1$	$\eta^*_\pi > 1$
$\eta_\pi > 1$	D	D, I
$\eta_\pi < 1$	D, I	I

$\sigma > 1, \sigma^* < 1$	$\eta^*_\pi < 1$	$\eta^*_\pi > 1$
$\eta_\pi > 1$	I	D, I
$\eta_\pi < 1$	D, I	D

$\sigma > 1, \sigma^* > 1$	$\eta^*_\pi < 1$	$\eta^*_\pi > 1$
$\eta_\pi > 1$	D, I	I
$\eta_\pi < 1$	D	D, I

D = determinate, I = indeterminate

In the case of $\left(\theta_1 \frac{\eta_y}{\eta_\pi} y_1 - \theta^*_2 \frac{\eta^*_y}{\eta^*_\pi} y^*_2\right) = 0$ and $\eta_y > 0$, "D" changes "D, I".

Table 2: Equilibrium determinacy if $\left(\theta_1 \frac{\eta_y}{\eta_\pi} y_1 - \theta_2^* \frac{\eta_y^*}{\eta_\pi^*} y_2^*\right) > 0$

$\sigma^* = 1$	$\sigma < 1$	$\sigma > 1$
$\eta_\pi > 1$	D, I	I
$\eta_\pi < 1$	I	D, I

$\sigma = 1$	$\sigma^* < 1$	$\sigma^* > 1$
$\eta_\pi^* > 1$	D, I	D, I
$\eta_\pi^* < 1$	D, I	D, I

$\sigma < 1, \sigma^* < 1$	$\eta_\pi^* < 1$	$\eta_\pi^* > 1$
$\eta_\pi > 1$	D, I	D, I
$\eta_\pi < 1$	D, I	D, I

$\sigma < 1, \sigma^* > 1$	$\eta_\pi^* < 1$	$\eta_\pi^* > 1$
$\eta_\pi > 1$	D, I	D, I
$\eta_\pi < 1$	D, I	D, I

$\sigma > 1, \sigma^* < 1$	$\eta_\pi^* < 1$	$\eta_\pi^* > 1$
$\eta_\pi > 1$	D, I	D, I
$\eta_\pi < 1$	D, I	D, I

$\sigma > 1, \sigma^* > 1$	$\eta_\pi^* < 1$	$\eta_\pi^* > 1$
$\eta_\pi > 1$	D, I	D, I
$\eta_\pi < 1$	D, I	D, I

D = determinate, I = indeterminate

Table 3: Equilibrium determinacy if $\left(\theta_1 \frac{\eta_y}{\eta_\pi} y_1 - \theta_2^* \frac{\eta_y^*}{\eta_\pi^*} y_2^*\right) < 0$

$\sigma^* = 1$	$\sigma < 1$	$\sigma > 1$
$\eta_\pi > 1$	D	I
$\eta_\pi < 1$	I	D

$\sigma = 1$	$\sigma^* < 1$	$\sigma^* > 1$
$\eta_\pi^* > 1$	D, I	I
$\eta_\pi^* < 1$	I	D, I

$\sigma < 1, \sigma^* < 1$	$\eta_\pi^* < 1$	$\eta_\pi^* > 1$
$\eta_\pi > 1$	D, I	D, I
$\eta_\pi < 1$	I	D, I

$\sigma < 1, \sigma^* > 1$	$\eta_\pi^* < 1$	$\eta_\pi^* > 1$
$\eta_\pi > 1$	D, I	D, I
$\eta_\pi < 1$	D, I	I

$\sigma > 1, \sigma^* < 1$	$\eta_\pi^* < 1$	$\eta_\pi^* > 1$
$\eta_\pi > 1$	I	D, I
$\eta_\pi < 1$	D, I	D, I

$\sigma > 1, \sigma^* > 1$	$\eta_\pi^* < 1$	$\eta_\pi^* > 1$
$\eta_\pi > 1$	D, I	I
$\eta_\pi < 1$	D, I	D, I

D = determinate, I = indeterminate

When η_y is positive, "D" changes "D, I".