Business Cycles and Exchange Rate Regimes

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RBC model with three countries

Steps:

1. Build a model with exchange rate fluctuations
2. Calibrate the model
3. Simulate the model economies
4. Compare the experimental moments with the data

   Does the model economy replicate the changes observed in the data?
5. If yes, what type of shock was responsible for these change?

Stylized facts:

- Volatilities are higher in North America (output, employment, trade balance, terms of trade
- Japanese aggregates tend to become less volatile
- European consumption becomes more procyclical, terms of trade more countercyclical
- Become more procyclical in North America: inverstment, exports, emplyment, imports. Countercyclical: terms of trade
- In Japan, employment become more procyclical, the terms of trade more countercyclical

The model

Consumer side

In each country, infinitely lived consumer, with intertemporal preferences over consumption and leisure:

$$\max_{\{c_{it}, n_{it}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t U(c_{it}, 1 - n_{it}) \right]$$

S.T.

$$E_0 \left[\sum_{t=0}^{\infty} \frac{w_{it} n_{it} + (r_{it} + \delta) k_{it}}{(1 + r_{it})^t} \right] = E_0 \left[\sum_{t=0}^{\infty} \frac{c_{it} + x_{it}}{(1 + r_{it})^t} \right]$$

with

$$U(c, 1 - n) = \frac{1}{\gamma} \left[c^{\mu} (1 - n)^{1-\mu} \right]^{\gamma}$$

For each agent a firm:

$$\max_{\{n_{it},k_{it}\}} z_{it}F(k_{it},n_{it}) - (r_{it}+\delta)k_{it} - w_{it}n_{it}$$

with

$$F(k,n) = k^{1-\theta}n^{\theta}$$

Use of production:

$$\alpha_i y_{it} = \alpha_i y_{iit} + \alpha_j y_{ijt} + \alpha_k y_{ikt}$$

Use of imports:

$$c_{it} + x_{it} = G(y_{iit}, y_{jit}, y_{kit})$$

where

$$G(y_1, y_2, y_3) = (\omega_1 y_1^{-\rho} + \omega_2 y_2^{-\rho} + \omega_3 y_3^{-\rho})^{-\frac{1}{\rho}}$$

Allows to introduce elasticity of substitution, $y_{ijt} > 0$ and $y_{jit} > 0$

Exchange rates:

Share π_i of imports billed in foreign currency

$$
\begin{aligned}
& E_0 \left[\sum_{t=0}^{\infty} \left(\frac{\alpha_j \left(\pi_j p_{it} + (1-\pi_j)\, p_{jt} e_{ijt}\right) y_{ijt}}{(1+r_{it})^t} + \right.\right. \\
& \qquad \left.\left. + \frac{\alpha_k \left(\pi_k p_{it} + (1-\pi_k)\, p_{kt} e_{ikt}\right) y_{ikt}}{(1+r_{it})^t} \right) \right] \\
= \quad & \alpha_i E_0 \left[\sum_{t=0}^{\infty} \left(\frac{\left(\pi_i p_{jt} e_{ijt} + (1-\pi_i)\, p_{it}\right) y_{jit}}{(1+r_{it})^t} + \right.\right. \\
& \qquad \left.\left. + \frac{\left(\pi_i p_{kt} e_{ikt} + (1-\pi_i)\, p_{it}\right) y_{kit}}{(1+r_{it})^t} \right) \right]
\end{aligned}
$$

Laws of motion:

Capital

$$k_{i,t+1} = (1-\delta)k_{i,t} + x_{it}$$

Investment projects

$$s_{j,t+1} = s_{j+1,t} \qquad j = 1, J-1$$

Technology innovations

$$\begin{pmatrix} z_{1,t+1} \\ z_{2,t+1} \\ z_{3,t+1} \end{pmatrix} = A_z(L) \begin{pmatrix} z_{1t} \\ z_{2t} \\ z_{3t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{z1,t+1} \\ \varepsilon_{z2,t+1} \\ \varepsilon_{z3,t+1} \end{pmatrix}$$

Exchange rates

$$\begin{pmatrix} e_{21,t+1} \\ e_{31,t+1} \end{pmatrix} = A_e(L) \begin{pmatrix} e_{21t} \\ e_{31t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{e21,t+1} \\ \varepsilon_{e31,t+1} \end{pmatrix}$$

Identities:

Investment

$$x_t = \sum_{j=1}^{J} \phi_j s_{jt}$$

Terms of trade

$$p_{jit} = \beta \frac{\partial G(y_{iit}, y_{jit}, y_{kit})}{\partial y_{jit}} \left[\frac{\partial G(y_{iit}, y_{jit}, y_{kit})}{\partial y_{iit}} \right]^{-1}$$

$$p^*_{it} = \frac{y_{jit} p_{jit} + y_{kit} p_{kit}}{y_{jit} + y_{kit}} \times$$

$$\frac{\alpha_j y_{ijt} + \alpha_k y_{ikt}}{\left((1-\pi_j) e_{ijt} \frac{p_{jt}}{p_{it}} + \pi_j\right) \alpha_j y_{ijt} + \left((1-\pi_k) e_{ikt} \frac{p_{kt}}{p_{it}} + \pi_k\right) \alpha_k y_{ikt}}$$

Trade balance

$$\left(\pi_i + (1-\pi_i) \frac{p_{jt}}{p_{it}} e_{ijt}\right) \frac{\alpha_j}{\alpha_i} y_{ijt} +$$
$$\left(\pi_i + (1-\pi_i) \frac{p_{kt}}{p_{it}} e_{ikt}\right) \frac{\alpha_k}{\alpha_i} y_{ikt} - p_{jit} y_{jit} - p_{kit} y_{kit}$$

The business cycle in this economy:

$$\begin{array}{l} \varepsilon_{zt} \longrightarrow z_t \longrightarrow \text{productivity} \\ \qquad\qquad\quad \nearrow \\ \varepsilon_{et} \longrightarrow e_t \longrightarrow p_t^* \end{array}$$

$$w_t \longrightarrow n_t$$

if shock is persistent:

$$w_t \longrightarrow c_t$$

$$r_t \longrightarrow c_t, x_t$$

$$p_t^*, x_t, c_t \longrightarrow y_{1t}, y_{2t}$$

$$z_t, n_t, x_{t-J} \longrightarrow y_t$$

Calibration:

Take some parameter values from the literature:

$$\beta, \delta, \rho, \theta, n, c, \gamma, \pi$$

Estimate some:

$$\frac{y_{ji}}{y_i}$$

$$\begin{pmatrix} z_{1,t+1} \\ z_{2,t+1} \\ z_{3,t+1} \end{pmatrix} = \begin{pmatrix} \overline{z}_1 \\ \overline{z}_2 \\ \overline{z}_3 \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} z_{1t} \\ z_{2t} \\ z_{3t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{z1,t+1} \\ \varepsilon_{z2,t+1} \\ \varepsilon_{z3,t+1} \end{pmatrix}$$

$$\begin{pmatrix} \varepsilon_{z1,t+1} \\ \varepsilon_{z2,t+1} \\ \varepsilon_{z3,t+1} \end{pmatrix} \rightsquigarrow \mathcal{N} \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} (\sigma_{z1})^2 & r_{z12}\sigma_{z1}\sigma_{z2} & r_{z13}\sigma_{z1}\sigma_{z3} \\ r_{z12}\sigma_{z1}\sigma_{z2} & (\sigma_{z2})^2 & r_{z23}\sigma_{z2}\sigma_{z3} \\ r_{z13}\sigma_{z1}\sigma_{z3} & r_{z23}\sigma_{z2}\sigma_{z3} & (\sigma_{z3})^2 \end{bmatrix} \right)$$

$$\begin{pmatrix} e_{21,t+1} \\ e_{31,t+1} \end{pmatrix} = \begin{pmatrix} \overline{e}_{21} \\ \overline{e}_{31} \end{pmatrix} + \begin{pmatrix} a_{e21} & 0 \\ 0 & a_{e31} \end{pmatrix} \begin{pmatrix} e_{21t} \\ e_{31t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{e21,t+1} \\ \varepsilon_{e31,t+1} \end{pmatrix}$$

$$\begin{pmatrix} \varepsilon_{e21,t+1} \\ \varepsilon_{e31,t+1} \end{pmatrix} \rightsquigarrow \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} (\sigma_{e21})^2 & r_e \sigma_{e21} \sigma_{e31} \\ r_e \sigma_{e21} \sigma_{e31} & (\sigma_{e31})^2 \end{bmatrix} \right)$$

Determine the others using the first order conditions.

Solution procedure:

Complex problem

Pareto Optimum = Market equilibrium

Quadratic approximation of the value fonction

Linear decision rules

Simulation with random numbers

Replication of stylized facts:

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Are these results stable?

- σ and π are critical for vol(trade): higher σ, lower π
- σ influences corr(output, invest) > corr(output, cons): lower σ
- idem for corr(output, imports) > corr(output, exports), also higher π
- corr(output, trade balance): lower σ
- corr(output, tot): lower σ, same π
- crosscorr(output) > crosscorr(cons): Hopeless?

Time-to-ship: prevents to much consumption smoothing

What now?

Find better estimates of σ and π

Endogenize the exchange rate movements

Cost of exchange rate movements