

**Technology Innovations  
and the Volatility of Output:  
An International Perspective**

Christian Zimmermann

How much of the business cycle can be explained by technology innovations measured by Solow residuals?

Kydland & Prescott (1982), Prescott (1986)

Focus:

- Look at other countries than the US
- Look at open economies

Opening an economy means:

- Risk sharing  $\Rightarrow$  less volatility in consumption and labor, ambiguous in output
- Additional external disturbances  $\Rightarrow$  more volatility in output

## RBC model with two countries

Steps:

1. Build the model, closed and open version
2. Calibrate to the individual countries' specificities
3. Simulate the model economies
4. Compare the artificial moments with the data
5. Consequences for future research

The model

Consumer side

Infinitely lived consumer, with intertemporal preferences over consumption and leisure:

$$\max_{\{c_t, n_t\}_{t=0}^{\infty}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - n_t) \right]$$

S.T.

$$w_t n_t + (r_t + \delta) q_t k_t + (1 + r_t) b_{t-1} = c_t + x_t + b_t$$

## Production side

Profit maximizing firm, using labor and capital inputs from households:

$$\max_{\{n_t, k_t\}} z_t F(k_t, n_t) - (r_t + \delta)q_t k_t - w_t n_t$$

$$z_t F(k_t, n_t) = y_{1t} + y_{2t}^*$$

Aggregation of domestic and foreign goods

$$c_t + x_t = A(y_{1t}, y_{2t})$$

Laws of motion

Capital

$$k_{t+1} = (1 - \delta)k_t + s_{1t}$$

Investment projects

$$s_{j,t+1} = s_{j+1,t} \quad j = 1, J - 1$$

Technology innovation

$$\begin{pmatrix} z_{t+1} \\ z_{t+1}^* \end{pmatrix} = A_z(L) \begin{pmatrix} z_t \\ z_t^* \end{pmatrix} + \begin{pmatrix} \varepsilon_{t+1} \\ \varepsilon_{t+1}^* \end{pmatrix}$$

Identities

Investment

$$x_t = \sum_{j=1}^J \phi_j s_{jt}$$

Trade balance

$$\alpha E_0 \left[ \sum_{t=0}^{\infty} p_t \frac{y_{2t}}{(1+r_t)^t} \right] = \alpha^* E_0 \left[ \sum_{t=0}^{\infty} p_t^* \frac{y_{2t}^*}{(1+r_t^*)^t} \right]$$

The business cycle in this economy:

$$\varepsilon_t \longrightarrow z_t \longrightarrow \text{productivity}$$

$$w_t \longrightarrow n_t$$

if shock is persistent:

$$w_t \longrightarrow c_t$$

$$r_t \longrightarrow c_t, x_t$$

$$x_t, c_t \longrightarrow y_{1t}, y_{2t}$$

$$z_t, n_t, x_{t-J} \longrightarrow y_t$$



Functional forms:

$$U(c, 1 - n) = \frac{1}{\gamma} [c^\mu (1 - n)^{1-\mu}]^\gamma$$

$$F(k, n) = k^{1-\theta} n^\theta$$

$$A(y_1, y_2) = (\omega_1 y_1^{-\rho} + \omega_2 y_2^{-\rho})^{-\frac{1}{\rho}}$$

$$\begin{pmatrix} z_{t+1} \\ z_{t+1}^* \end{pmatrix} = \begin{pmatrix} \bar{z} \\ \bar{z}^* \end{pmatrix} + \begin{pmatrix} a_1 & a_2 \\ a_1^* & a_2^* \end{pmatrix} \begin{pmatrix} z_t \\ z_t^* \end{pmatrix} + \begin{pmatrix} \varepsilon_{z,t+1} \\ \varepsilon_{z,t+1}^* \end{pmatrix}$$

$$\begin{pmatrix} \varepsilon_{z,t+1} \\ \varepsilon_{z,t+1}^* \end{pmatrix} \rightsquigarrow \mathcal{N} \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} (\sigma_z)^2 & r_z \sigma_z \sigma_z^* \\ r_z \sigma_z \sigma_z^* & (\sigma_z^*)^2 \end{bmatrix} \right)$$

$$z_{t+1} = \bar{z} + \rho_z z_t + \varepsilon_{z,t+1}$$

$$\varepsilon_{z,t+1} \rightsquigarrow \mathcal{N}(0, \sigma_z^2)$$

## Calibration

### Global parameters

- labor  $n = 0.3$
- risk aversion / intertemporal substitution  
 $\gamma = -1$
- time to build  $J = 3$
- project share  $\phi_j = \frac{1}{3}, j = 1, J$

## Country specific parameters

- labor income share  $\theta$ , from .547 to .649
- import share  $\frac{y_2}{y}$ , from .083 to .507
- capital output ratio  $\frac{k}{y}$ , from 3.79 to 9.54
- investment output ratio  $\frac{x}{y}$ , from .206 to .368
- relative size  $\alpha$ , from .005 to .391

Elasticity of substitution:  $\sigma = \frac{1}{1+\rho}$

$$A(y_1, y_2) = (\omega_1 y_1^{-\rho} + \omega_2 y_2^{-\rho})^{-\frac{1}{\rho}}$$

- import share  $\frac{y_2}{y_1+y_2}$  has risen;
- tariffs  $t$  have decreased;
- transportation costs  $\tau$  have decreased;
- terms of trade  $p$  have changed.

$$\frac{y_2}{y_1} = \left(\frac{\omega_1}{\omega_2}\right)^{-\frac{1}{\rho+1}} \left(p \frac{1+t+\tau}{1-\tau}\right)^{-\frac{1}{\rho+1}}$$

$$\rho = \frac{\log\left(\frac{1-\tau_1}{1+t_1+\tau_1}\right) - \log\left(\frac{1-\tau_0}{1+t_0+\tau_0}\right) - \log\left(\frac{p_1}{p_0}\right)}{\log\left(\frac{y_{2,1}}{y_{1,1}}\right) - \log\left(\frac{y_{2,0}}{y_{1,0}}\right)} - 1$$

## Solow residuals

$$z_t = \frac{y_t}{n_t^\theta} \quad z_t = \frac{y_t}{(n_t h_t)^\theta}$$

$$z_t = \frac{y_t}{k_t^{1-\theta} n_t^\theta} \quad z_t = \frac{y_t}{k_t^{1-\theta} (n_t h_t)^\theta}$$

$$\begin{pmatrix} z_{t+1} \\ z_{t+1}^* \end{pmatrix} = \begin{pmatrix} \bar{z} \\ \bar{z}^* \end{pmatrix} + \begin{pmatrix} a_1 & a_2 \\ a_1^* & a_2^* \end{pmatrix} \begin{pmatrix} z_t \\ z_t^* \end{pmatrix} + \begin{pmatrix} \varepsilon_{z,t+1} \\ \varepsilon_{z,t+1}^* \end{pmatrix}$$

$$\begin{pmatrix} \varepsilon_{z,t+1} \\ \varepsilon_{z,t+1}^* \end{pmatrix} \rightsquigarrow \mathcal{N} \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} (\sigma_z)^2 & r_z \sigma_z \sigma_z^* \\ r_z \sigma_z \sigma_z^* & (\sigma_z^*)^2 \end{bmatrix} \right)$$

$$z_{t+1} = \bar{z} + \rho_z z_t + \varepsilon_{z,t+1}$$

$$\varepsilon_{z,t+1} \rightsquigarrow \mathcal{N}(0, \sigma_z^2)$$

## Conclusions

Opening an economy adds  $\sim 10\%$

Technology shocks **can** explain all the volatility of output

The United States are a “special case”

Measurement issues:

- Measurement error
- Definition of Solow residuals matters
- Solow residuals may not be the appropriate measure

A reorientation of research is necessary