Supplementary Material

Density of $Y = \sqrt{1 + X^2}$

Cumulative distribution of Y:

$$\begin{aligned} P(Y < y) &= P(|X - \mu| < \sqrt{y^2 - 1}) && (61) \\ &= P(-\sqrt{y^2 - 1} < X - \mu < \sqrt{y^2 - 1}) && (62) \\ &= P(-\sqrt{y^2 - 1} + \mu < X < \sqrt{y^2 - 1} + \mu) && (63) \\ &= F_X(\sqrt{y^2 - 1} + \mu) \quad - (1 - F_X(\sqrt{y^2 - 1} - \mu)). && (64) \end{aligned}$$

Probability density of Y:

$$\begin{aligned} p_Y(y) &= \frac{\mathrm{d}}{\mathrm{d}y} P(Y < y) && (65) \\ &= \frac{\mathrm{d}}{\mathrm{d}y} \left[F_X(\sqrt{y^2 - 1} + \mu) - (1 - F_X(\sqrt{y^2 - 1} - \mu)) \right] && (66) \\ &= \frac{1}{\sqrt{2\pi}\sigma} \left[\exp\left(-\frac{(\sqrt{y^2 - 1} + \mu)^2}{2\sigma^2} \right) + \exp\left(-\frac{(\sqrt{y^2 - 1} - \mu)^2}{2\sigma^2} \right) \right] \frac{y}{\sqrt{y^2 - 1}}. \end{aligned}$$

Mean of Y:

$$\begin{aligned} \mathbb{E}_{p_Y(y)}[y] &= \int_1^\infty y \; p_Y(y) \mathrm{d}y && (67) \\ &= \frac{1}{\sqrt{2\pi}\sigma} \int_1^\infty \left[\exp\left(-\frac{(\sqrt{y^2 - 1} + \mu)^2}{2\sigma^2} \right) + \exp\left(-\frac{(\sqrt{y^2 - 1} - \mu)^2}{2\sigma^2} \right) \right] y^2 (y^2 - 1)^{-\frac{1}{2}} \mathrm{d}y. && (68) \end{aligned}$$

At first glance this looks to be an intractable integral, however, with the change of variables $y = (x^2 + 1)^{1/2}$ and by expanding the exponential cross terms we arrive at:

$$\mathbb{E}[y] = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\mu^2}{2\sigma^2} \right) \sum_{l=0}^{\infty} \frac{\Gamma\left(l + \frac{1}{2}\right)}{(2l)!} \left(\frac{\mu}{\sigma^2} \right)^{2l} U\left(l + \frac{1}{2}, l + 2, \frac{1}{2\sigma^2} \right). \qquad (69)$$