Anwesha Banerjee
Stefano Barbieri
Kai A. Konrad

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Department of Business and Tax Law
Department of Public Economics

http://www.tax.mpg.de

Max Planck Institute for Tax Law and Public Finance
Marstallplatz 1
D-80539 Munich
Tel: +49 89 24246 – 0
Fax: +49 89 24246 – 501
E-mail: ssrn@tax.mpg.de
http://www.tax.mpg.de

Climate Policy, Irreversibilities and Global Economic Shocks

Anwesha Banerjee*, Stefano Barbieri† and Kai A. Konrad‡

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Abstract

Global systematic economic shocks may affect the Nash equilibrium contributions to international climate mitigation. We study how this effect depends on the flexibility countries have to adjust to these shocks. The kind of rigidities countries face because of technological irreversibilities plays a crucial role. Under the plausible assumption of "prudence," higher global uncertainty tends to reduce equilibrium climate contributions if irreversibilities in the *level* of climate policy choices exist. And, if countries are committed to allocating a *proportion* of income to climate protection, rigidities may increase welfare. Thus, exercising the option to perfectly adjust one's contributions to shocks may be another form of free riding.

JEL classification codes: Q54, H41, Q55

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1 Introduction

We study how global systematic prosperity risks affect individual countries' non-cooperative contributions to climate protection under several distinguishable regimes of technological and investment irreversibility. Several events in the early decades of the 21st century showed that the world is exposed to risks that make economic prosperity in terms of world income a random variable. Recent examples are the great financial crisis, the pandemic crisis, and major warfare

*Max Planck Institute for Tax Law and Public Finance, Marstallplatz 1, D-80539 Munich, Germany, E-mail: Anwesha.Banerjee@tax.mpg.de.

†Department of Economics, 206 Tilton Hall, Tulane University, New Orleans, LA 70118, United States. E-mail: sbarbier@tulane.edu.

‡Corresponding Author. Max Planck Institute for Tax Law and Public Finance, Marstallplatz 1, D-80539 Munich, Germany, phone: +49 89 24246-5251, Germany. Orcid ID: 0000-0002-2225-2271. E-mail: kai.konrad@tax.mpg.de.

and its economic implications.[1] As we describe further below, major technological or investment choices made by countries may lead to irreversibilities and limit countries' ability to react flexibly to such global events, particularly with respect to their contributions to global climate policy.[2] We ask how the prosperity shocks in conjunction with such irreversibilities affect the countries' ability and motivation to contribute to climate policy in a Nash contribution game.[3]

To elaborate on the types of rigidies, think of a country that might decide how many power plants for gas, oil, coal or nuclear energy to construct or to close down, or whether to create an infrastructure based on e-mobility, invest in green energy, etc. The fact that some technological choices may bind for a long time is well-recognized.[4] The empirical relevance of rigidities in their climate policy has been discussed in the context of choice of countries' energy industry. Actual climate policy might need long-lasting and binding decisions that are taken when the course of the economy is still uncertain. Baldwin, Cai and Kuralbayeva (2020) discuss irreversible investment both for the "clean" and a "dirty" energy sector. Pfeiffer, Millar, Hepburn and Beinhocker (2016) discuss inertias emerging from the remaining lifetime of the stock of thermal power plants. Fouquet (2016) discusses path-dependence and technological lock-ins in a country's energy sector. Erickson, Kartha, Lazarus and Tempest (2015) make an attempt to quantify the "carbon lock-in", that is, the amount of emissions that are basically pre-determined by long-term technological and physical investment choices. "Technical equipment lifetime" is one of the reasons for inertia they consider and they refer to coal-power equipment with lifetimes between about 40 and 50 years. Choices of technology and capacity investment may often lead to commitment about the amount of feasible emission reduction for decades into the future.[5]

[1] A list of major global risks and how experts assess their probability and impact is in the World Economic Forum's *Global Risks Report* (World Economic Forum 2021). According to Baker, Bloom and Davis (2016) (who develop the World Uncertainty Index, an index of economic policy uncertainty based on newspaper coverage frequency), global uncertainy levels have been rising since the 1960s. Ahir, Bloom and Furceri (2021) report that global uncertainty levels as measured by the World Uncertainty Index, though declining post the coronavirus crisis, remain high.

[2] On a most general level, Pindyck (1991) alluded to the important role of irreversibility of many types of investment in the context of firms and to the option value of delayed choice that reacts to new information.

[3] It has also been noted that major prosperity shocks can affect the attention given to the problem of global warming. For example, Botzen, Duijndam, and van Beukering (2021) and Evensen, Witmarsh, Bartie, et al. (2021) point to a crowding out of awareness of impending risks from the climate change problem. Our analysis is not about attention issues, but about equilibrium behavior among fully rational and well-informed governments.

[4] Pizer and Kopp (2005, p. 1318), for example, refer to Dixit and Pyndick (1994) and Pindyck (1995) and state: "The potential exists for environmental protection activities to involve irreversible costs [so that] the cost of a policy is raised by the forgone option value associated with waiting."

[5] As Pindyck (1995, p.2) puts it: "First, policies aimed at reducing ecological damage impose sunk costs on society. These sunk costs can take the form of discrete investments; for example, coal burning utilities might be forced to install scrubbers, or firms might have to scrap existing machines and invest in more fuel-efficient ones. Or they can take the form of flows of expenditures, e.g., a price premium paid by a utility for low-sulfur coal. In either

In the most flexible case, a country can wait for future economic conditions to realize and then make an individual optimizing choice based on known national income. However, choosing an allocation after observing whether a major shock occurred might not be feasible. Rigidities caused by irreversible technology choices might bind the country to a particular path that is chosen before uncertainty is resolved. Different degrees and types of rigidity may affect the strategic interaction that emerges if countries make independent choices about their contribution to worldwide climate protection. We consider four possible hypothetical scenarios. One is characterized by full ex-post flexibility in making use of the country's GDP. A second one is characterized by a fixed amount of emission reduction independent of economic prosperity, implying that any income shock is absorbed by the amount of national non-climate related expenditure. A third one fixes national "private" consumption and makes climate protection the residual variable. A fourth case fixes the relative shares of future national income used for national consumption and climate policy, making the relative shares independent of the prosperity performance. As countries are typically subject to similar constraints that guide their technology choices and their carbon lock-in, the form of the rigidity is likely to be similar in the different countries. We analyze the respective Nash equilibria on climate protection under the four different forms of rigidity and determine the effects of global income uncertainty. We also compare the equilibrium expected quantity of the contributions to climate policy and the equilibrium welfare under the different forms of rigidity.

A key finding of the paper is that, under prudence, total public good contributions are reduced by technological constraints that compel a country to set aside contributions to the public good before the income uncertainty is resolved. That is, if reducing carbon emissions needs upfront long-term investments, countries acting independently will, in sum, contribute less to emissions reduction than if there were no such constraints. Another result of the paper shows that, under the alternative constraint for which countries must allocate a certain proportion of their income to the public good (instead of a fixed contribution amount), rigidities may improve welfare. To understand this result, note that, as an implication of Pindyck's (1991) considerations on irreversibility and the option value of flexibility, a country that makes stand alone choices does best if it can wait for future economic conditions to realize and then make an optimizing choice based on known national income. But it is not clear that this advantage of flexibility remains valid in a non-cooperative Nash equilibrium of voluntary contributions. The distortions caused by rigidities may counteract the incentives to free ride if all countries face rigidities, so the effect on welfare is not obvious.

Our paper is related to the abstract theory of voluntary provision of a public good that emphasizes uncertainty and restrictions on the countries' strategy set. Though many different sources of uncertainty have been considered in this

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case, such sunk costs create an opportunity cost of adopting a policy now, rather than waiting for more information about ecological impacts and their economic consequence."

context, there is limited research on the consequences of group-wide uncertain future endowments and on decision rigidities. Some early papers like Dardanoni (1988) and Gradstein, Nitzan, and Slutsky (1992) focus on the general case of uncertainty faced by individuals when their utility function depends upon two goods. These papers are unconcerned with the implications of uncertainty for public good provision, though they cite private provision as an application of their framework. Gradstein, Nitzan, and Slutsky (1993) is one of the first papers in this literature to look at the situation where the price of the public good provided is subject to uncertainty. Other papers address the consequences of uncertainty regarding the contributions of other countries. In Cornes and Sandler (1984) and Sandler, Sterbenz, and Posnett (1987), countries are uncertain about what others are contributing, and may form beliefs about how their contributions affect the contributions of others.[6] Keenan, Kim and Warren (2006) study the symmetric Nash equilibrium under the same type of uncertainty, in contrast to the previous papers that focused on the single player's problem. A third source of uncertainty is incomplete information regarding fellow players. This kind of situation can arise due to countries having heterogeneous preferences for the public good, and beliefs about what the preferences of other countries may be (Maldonado and Neto (2016)).

Incomplete information may also play a role when countries' valuations of the public good are private information, such as first studied by Gradstein, Nitzan, and Slutsky (1994) and more recently by Barbieri and Malueg (2008). Bac (1996) complements this literature with an analysis of a dynamic game where agents (countries) have private valuations for international environmental resources. While there is a large literature that explores the topic of private valuations of public goods,[7] none of the papers mentioned so far consider the type of uncertainty arising because future incomes are unknown due to aggregate income uncertainty.

The analysis in Robledo (1999) comes closest to the one in the current paper, in that he examines the decisions of two countries with uncertain future incomes who must make a decision about their contribution to a public good before this uncertainty is resolved. This corresponds to the "fixed contributions" case of our model. His focus, however, is on uncertainty as a variable, and the strategic role of higher or lower uncertainty in one player's endowment for own and others' contributions in the equilibrium.

This paper indirectly also contributes to the literature in environmental economics on the role of uncertainty in helping or hindering solutions to global commons problems. Kolstad (2005), Kolstad (2007) and Ulph (2004), study the formation of international environmental agreements (IEAs) when there is uncertainty about the environmental costs of pollution abatement. They assume that countries are risk-neutral. In contrast, Bramoullé and Boucher (2010) consider the case when countries are risk-averse and find that uncertainty may

[6] See also Cornes and Sandler (1985), Shogren (1987), Shogren (1990) and Sugden (1985).

[7] See Bag and Roy (2008), Bag and Roy (2011) for sequential settings, Barbieri and Malueg (2010) and Menezes et al. (2001) for discrete public goods, and Barbieri and Malueg (2016) for a comparison of the discrete and continuous public good settings, among others.

increase the number of signatories to IEAs. Bramoullé and Treich (2009) study the setting where individuals face the same risk of damage from pollution and find that uncertainty reduces pollution emissions and improves individuals' welfare. Bochet et al. (2019) study a modified Nash demand game where agents can demand shares of a common pool resource, and the threshold level beyond which the amount of the resource available collapses to zero is subject to uncertainty. Kuusela and Laiho (2020) model a two-stage game where countries choose to strategically acquire information regarding the damages from pollution in the first stage, when this damage is uncertain, before committing to pollution abatement in the second stage. Schumacher (2015), and more recently, Banerjee and Gravel (2020) look at how individuals' beliefs may affect their contributions to a public good when there is uncertainty. Auerswald, Konrad and Thum (2018) take account of the uncertainty about the impact of climate change and allow countries to choose mitigation effort as well as adaptation effort. Also a strand of the literature studies the global climate problem in the face of incomplete information and how institutional provisions affect the likelihood of reaching an agreement. Konrad and Thum (2014) study the role of unilateral commitment on environmental policies when countries negotiate about a cooperative environmental agreement if incomplete information prevails, showing that unilateral commitment reduces the probability for reaching an agreement. In Konrad and Thum (2018) they study the effect of the Clean Development Mechanism that was an essential element of the Kyoto Protocol for the chances of reaching a cooperative environmental agreement. Our analysis of the determinants of non-cooperative agreement is also relevant for research on cooperative outcomes, as the non-cooperative equilibrium is the default of such cooperative agreement and therefore affects the bargaining game between the countries.

We proceed as follows. Section 2 formally presents the framework of the four different technological constraints that countries may face. Section 3 derives the equilibrium outcomes of the four games. Section 4 presents a comparison of expected levels of the public good in each of the cases studied and some implications for ex-ante welfare. Section 5 concludes.

2 A formal framework

Consider an uncertain world with n symmetric countries $i \in \{1, ..., n\} = \mathcal{N}$. The assumption of symmetry is important. It allows us to abstract from possible differences between countries in terms of size, risk-attitudes, preferences, technology constraints etc. in order to highlight how, under income uncertainty, different possible technological frictions on countries affect the risk-allocation in the voluntary provision game.[8] Uncertainty is about the state of the world. There are S states of the world, indexed by $s \in \{1, ..., S\} = \mathcal{S}$. Probabilities p_s are assigned to the states. Each country has an income that is state-dependent and equal to m_s^i, where symmetry means $m_s^i = m_s$ for all $i \in \mathcal{N}$. The value of

[8] Symmetry also removes issues of distributional effects from the picture that would be interesting to study, but orthogonal to the questions addressed here.

m_s might be seen as the random output based on the production facilities of the country and a global systematic random component, such as uncertain universal technological progress, major global crises such as the financial crisis or the pandemic crisis, or other major global risks that are looming and affect the path of economic development and general prosperity. We number states such that $\min_{s\in\mathcal{S}} m_s = m_1$, i.e., the state with the smallest income is state $s = 1$. Countries know m_s for each state and the probability distribution $(p_1, ..., p_S)$. Let average income be $\overline{m} = \sum_{s\in S} p_s m_s$.We consider several games, and these differ in whether countries know the true state s before making decisions.[9]

Countries use part of their incomes for making contributions to world climate protection, which is the global pure public good. The amount of income not contributed is consumed in the respective country and has no international spillover. Drawing on the theory of non-cooperative voluntary contributions to a public good we refer to it as the country's "private consumption." Countries' payoffs are measured in expected utility (of their representative citizen). Utility is additively separable in the two goods: in a given state,

$$U_s^i = u(m_s - g_s^i) + v(G_s), \tag{1}$$

where g_s^i is the contribution of country i to the public good and G_s is the aggregate amount of the public good that emerges from the individual contributions, assuming an additive aggregation function

$$G_s = \sum_{i\in\mathcal{N}} g_s^i. \tag{2}$$

As is commonly assumed, u and v are continuously differentiable, strictly increasing and strictly concave. If u and v differ in their curvature, then the two consumption components differ in their risk-absorbing capacities and their comparison has important consequences for our analysis. Furthermore, we will often assume that u and v display "prudence," i.e., $u''' > 0$, $v''' > 0$.[10] We assume that countries maximize their own expected utility

$$\sum_{s\in\mathcal{S}} p_s(u(m_s - g_s^i) + v(G_s)). \tag{3}$$

Finally, in all our analysis we characterize and focus on interior solutions for all countries.

We study four games that differ in the definition of countries' action spaces.

[9] The focus on global risks is important. As is well-known, idiosyncratic risks are largely eliminated out in the model of voluntary contributions. This is an implication of Warr (1983).

[10] In the framework of expected utility as per Von Neumann and Morgenstern (1944), the concept of higher-order risk attitudes was first introduced by Leland (1968), and subsequently formalized as prudence by Eeckhoudt and Kimball (1992) and Kimball (1990) to explain precautionary savings. A prudent individual is characterized by a positive third derivative of the utility function. While less common than risk-aversion, prudence is a widely-accepted measure of precautionary attitudes even outside of savings behavior, and can be understood as characterizing a decision maker who prefers to combine both "good and bad outcomes" rather than face "all good" or "all bad" outcomes (Eeckhoudt, Laeven, and Schlesinger (2020)).

Game 1 (Full ex-post flexibility). Each country observes m_s and chooses the amount $g_s^i \in [0, m_s]$ of contribution, which implicitly also determines $x_s^i = m_s - g_s^i$: the private consumption in this country in this state of the world. This framework is closest to the standard problem of voluntary provision of a public good, and serves as a benchmark.

Game 2 (Fixed absolute contributions to climate protection). Each country can pledge a state-independent future contribution $g^i \in [0, m_1]$ to the public good (recall: $\min_{s \in \mathcal{S}} m_s = m_1$ by appropriate state numbering). Negative contributions are not feasible, and contributions cannot exceed the country's income. National consumption in the country is then $x_s^i = m_s - g^i$. It is the residual of national income, and absorbs all national income risk. For illustration, we might think of a situation in which contributions to climate policy require technology choices that have a long-lasting, irreversible impact. As briefly discussed, the choice of the energy mix might be such a choice, as the decisions of replacing power plants that rely on carbon or fossil oil and gas by green energy, or the switch in the transport sector from combustion engines to electric or hydrogen-based engines have long-lasting commitment effects.

Game 3 (Fixed private consumption). Each country can make expenditure commitments on future private consumption $x^i \in [0, m_1]$ prior to learning the state of the world.. The interval of possible x^i accounts for the non-negative values of both a country's contribution to the public good as well as the country's private consumption. Game 3 is the mirror image of Game 2. Private consumption and contributions to climate protection switch roles: future national consumption becomes an income-independent quantity and all income-risk bearing is placed on contributions to climate policy. Motivating examples for this case are less obvious, as institutional or technological constraints that fix private consumption and make it impossible for private consumption to react to national income are much less plausible, given the variety of items of private consumption. We include this case for completeness as an interesting theory comparison.

Game 4 (Fixed proportional contributions). Each country can pledge the contribution $\gamma^i m_s$ to the public good, and this contribution share γ^i is independent of the state of the world. Contribution shares are from the interval $\gamma^i \in [0, 1]$. This scenario is somewhere between the three other scenarios. It allows the country to react to high or low national income along both dimensions, much like case 1, but in fixed proportions. Much like in Games 2 and 3, the possible reactions to national income levels are not completely independent. In Game 1 the country can react in a way that is fully individually optimal. In Game 4 both types of utility-generating expenditure adjust to higher or lower income, but the reactions are tied to each other by strict proportionality. As we briefly discussed in Footnote 5, a motivating example may be a contract that today binds a country to pay a price premium for a less polluting input whose use is directly proportional to income.

The above completes the descriptions of four variants of a voluntary contribution game in the presence of global/systematic output uncertainty. Next we solve for the equilibria in these games and compare them. The Nash equilibrium in all four games will be generically inefficient, given the positive externality of contributions. But the constraints of how countries can adjust to the low or high budget make it less clear what the overall effects for expected public-good provision and welfare will be.

3 Equilibrium for the four games

Full flexibility We start with a benchmark-setup (*Game 1*). Each country can observe the state of the world, forms expectations about what the contributions of the other $n-1$ countries will be, and chooses own contributions as an optimal reply to these Nash conjectures. As states of the world are mutually exclusive events, any state that materializes leads to a standard public-good contribution game. Let the state of the world be s. The fact that another state $s' \neq s$ could have emerged is irrelevant for the choices and payoffs emerging in s. Formally, country i solves the problem of maximizing (1), for all possible states, resulting in S Nash levels of the public good $G_1, ..., G_S$. In a symmetric equilibrium, $g_s^i = G_S/n$ for any $i \in N$.

Proposition 1 *The condition that determines the interior symmetric equilibrium provisions of the public good and individual countries' private consumptions for Game 1 is*

$$u'(m_s - \frac{1}{n}G_s) = v'(G_s) \text{ for all } s = 1, ..., S. \tag{4}$$

This equilibrium exists for state s if $u'(0) > v'(nm_s)$ and $v'(0) > u'(m_s)$, for all s. When this symmetric equilibrium exists, it is the unique interior equilibrium.

Proof. The equilibrium analysis can be carried out for each s independently. For each state s the equilibrium outcome is a special parametric case of Bergstrom, Blume and Varian (1986) and Buchholz (1990) for which existence and uniqueness is well known. For an interior symmetric equilibrium, (4) is the first-order condition. Therefore, if we show existence and uniqueness of a solution to (4), we have also shown that there exists a unique equilibrium, which happens to be symmetric. The left-hand side of (4) is continuous and strictly increasing in $G \in [0, nm_s]$ and the right-hand side of (4) is continuous and strictly decreasing in $G \in [0, nm_s]$. Accordingly, this interior equilibrium exists for state s if the left-hand side is smaller than the right hand side for $G = 0$ (i.e., $u'(m_s) < v'(0)$) and if the left-hand side is strictly larger for $G = nm_s$ (i.e., $u'(0) > v'(nm_s)$). ■

Multiplying the S marginal conditions by the probabilities of the different states of the world and summing them up yields, using symmetry,

$$\sum_{s \in \mathcal{S}} p_s u'(m_s - \frac{1}{n}G_s) = \sum_{s \in \mathcal{S}} p_s v'(G_s), \tag{5}$$

which states that, overall, the individual country has the same expected marginal utility from private and public good consumption. Note that the condition (5) reduces to the optimal allocation choice of a player who makes optimal consumption choices in a two-dimensional consumption space for a budget m_s, equalizing marginal utility for each good in this state. If this occurs for all states of the world, then this also equalizes expected utility of a marginal unit of expenditure on the two consumption items. The public good problem simply adds to this in the sense that other players also contribute an anticipated amount to the public good, and for a given player in a given state s this anticipated contribution sum is simply an expansion of the own budget by an additional amount $\sum_{j \in \mathcal{N} \setminus \{i\}} g_s^i$, and so for all independent problems $s = 1, ..., S$.

Game 1 is the benchmark case used to study the three types of technological constraints which we look at now.

Fixed contributions to climate protection Suppose countries have to decide on their contributions to the public good before knowing the realization of the state of the world. Recall that this describes a situation in which climate policy choices on technology have a long-lasting and irreversible impact, such as choices whether to close down (or not start building) coal-burning power plants, to rely on nuclear power plants or on wind- and solar energy, investing in "green" steel production using hydrogen, or transforming the mobility sector from combustion energy to electric or other types of climate friendly technology (changing the car fleet, building up the network of electric charging stations and of long-distance power lines). Once g^i is chosen before s is known, the private consumption in the country becomes the residual $x_s^i = m_s - g^i$, and differs for the different income states $s = 1, ..., S$. In a symmetric equilibrium, $g^i = G/n$ for any $i \in N$, where G is the equilibrium total public good provided. The following describes the contribution equilibrium:

Proposition 2 *The condition that characterizes the unique symmetric interior equilibrium provisions of the public good and individual countries' consumption for Game 2 is*

$$\sum_{s \in \mathcal{S}} p_s u'(m_s - \frac{G}{n}) = v'(G). \tag{6}$$

This equilibrium exists if $\sum_{s \in \mathcal{S}} p_s u'(m_s) < v'(0)$ *and* $\sum_{s \in \mathcal{S}} p_s u'(m_s - m_1) > v'(nm_1)$. *When this symmetric equilibrium exists, it is the unique interior equilibrium.*

Proof. We first characterize a symmetric equilibrium, then show existence and uniqueness of this equilibrium among symmetric equilibria, and then show that an asymmetric equilibrium cannot exist. For symmetric equilibrium it must hold that $g^i = G/n$ for all $i = 1, ..., n$. Hence, the first-order condition for

$$\max_{g^i} \sum_{s \in \mathcal{S}} p_s u(m_s - g^i) + v(\sum_{j \in \mathcal{N}} g^j)$$

can be written as (6). Recall $m_1 = \min_{s\in\mathcal{S}}\{m_s\}$, such that feasible quantities of the public good are in the range $G \in [0, nm_1]$. The left-hand side of (6) is a continuous and strictly monotonically increasing function in G. The right-hand side of (6) is a continuous and strictly monotonically decreasing function in G. This equation therefore has one and only one solution for interior $G \in (0, nm_1)$ if $\sum_{s\in\mathcal{S}} p_s u'(m_s) < v'(0)$ (left-hand side smaller than right-hand side at $G = 0$) and $\sum_{s\in\mathcal{S}} p_s u'(m_s - m_1) > v'(nm_1)$ (left-hand sider larger than right-hand side at $G = nm_1$). It remains to show that an interior equilibrium, if it exists, has to be symmetric. Suppose $G \in (0, nm_1)$ is the equilibrium quantity of the public good in an interior equilibrium. Note that, by construction of Game 2, country i's choice of g^i is the same for all states s. Suppose now $g^i > g^{i'}$ for some i and i'. Then $x_1^i < x_1^{i'}$, and as $x_s^i = x_1^i + (m_s - m_1)$, it follows that $x_s^i < x_s^{i'}$ for all $s \in S$. Accordingly,

$$\sum_{s\in\mathcal{S}} p_s u'(x_s^i) > \sum_{s\in\mathcal{S}} p_s u'(x_s^{i'}). \tag{7}$$

Optimality of these x_1^i and $x_1^{i'}$ requires that

$$v'(G) = \sum_{s\in\mathcal{S}} p_s u'(x_s^i) \text{ and} \sum_{s\in\mathcal{S}} p_s u'(x_s^{i'}) = v'(G), \tag{8}$$

and (7) and (8) are incompatible with each other. This completes the proof. ■

Intuitively, like in the standard public good game, countries' contributions to the public good generate direct public good utility and reduce private-good consumption utility. While without uncertainty this utility reduction is $-u'(m_s + G^{-i} - G)dG$, it becomes now $(\sum_{s\in\mathcal{S}} p_s u'(m_s + G^{-i} - G))dG$. But this expectation operator inherits the concavity properties and the additive separability, so from a formal point of view the machinery of Bergstrom, Blume and Varian (1986) (or the more elegant framework by Buchholz (1990)) is unchanged. We also note that each component x_s^i of this vector is fully informative for all its components: for instance, a given private consumption level in state s implies that the private consumption level in state s' is higher (lower) by the amount $(m_{s'} - m_s)$. The way the different components of state-contingent consumption are tied to each other reduces the problem to a standard choice, much like in Bergstrom, Blume and Varian (1986), where the marginal utility of private consumption is replaced by marginal expected utility of private consumption and where the country has only one degree of freedom of choice.

It is interesting to assess the implications of riskiness of income in this context.

Proposition 3 *Let $u''' > 0$ (prudence). In Game 2 (fixed public good contributions) systematic income risk reduces the equilibrium provision to the public good compared to certain income.*

Proof. Let $\bar{G}$ be the equilibrium quantity of the public good under certainty. We note:

$$v'(\bar{G}) = u'(\bar{m} - \frac{\bar{G}}{n}) = u'(\sum_{s\in\mathcal{S}} p_s(m_s - \frac{\bar{G}}{n})).$$

The first equation is the equilibrium first-order condition, and the second equation is a straightforward transformation. Now:

$$u'(\sum_{s\in\mathcal{S}} p_s(m_s - \frac{\bar{G}}{n})) < \sum_{s\in\mathcal{S}} p_s u'(m_s - \frac{\bar{G}}{n})$$

if $u''' > 0$ (prudence) by Jensen's inequality. Accordingly,

$$v'(\bar{G}) < \sum_{s\in\mathcal{S}} p_s u'(m_s - \frac{\bar{G}}{n}). \tag{9}$$

Let the equilibrium public good quantity in Game 2 be G_F and note that it solves (6), i.e.,

$$v'(G_F) = \sum_{s\in\mathcal{S}} p_s u'(m_s - \frac{G_F}{n}). \tag{10}$$

Visual inspection of (9) and (10) reveals that G_F must have a larger left-hand-side and a smaller right-hand side. To formally prove this claim that $G_F < \bar{G}$, assume by contradiction that $G_F \geq \bar{G}$. We then have by v' decreasing and (10) that

$$v'(\bar{G}) \geq v'(G_F) = \sum_{s\in\mathcal{S}} p_s u'(m_s - \frac{G_F}{n}).$$

Now, by u' is decreasing and $G_F \geq \bar{G}$ we have

$$\sum_{s\in\mathcal{S}} p_s u'(m_s - \frac{G_F}{n}) \geq \sum_{s\in\mathcal{S}} p_s u'(m_s - \frac{\bar{G}}{n}).$$

Finally, by (9) we have

$$\sum_{s\in\mathcal{S}} p_s u'(m_s - \frac{\bar{G}}{n}) > v'(\bar{G}),$$

and the extremes of the chain of the last three displayed inequalities are incompatible. ■

Intuitively, if there is no income risk, then both private and public good consumption components are certain. If income becomes systematically uncertain, then it becomes important that technological commitment to an income-independent contribution to the public good shifts all this risk to the private-consumption utility component. Countries make provisions for this risk and take precautions. If they are 'prudent' in the private consumption utility component (convexity of u'), this makes them shift resources from the safe use to the risky utility component. The equilibrium contributions to the public good are smaller under uncertainty than under certainty.

Fixed national private consumption choices Let us now reverse the assumptions about which type of consumption is less flexible due to long-term technology irreversibility. As private consumption x^i is a 'catch all' category consisting of a large basket of goods that include components for which consumption is quite flexible also in the short run, a fixed, state-independent x^i is less well motivated than the assumption that g^i is fixed. However, for completeness and comparison, suppose countries must choose fixed levels of their private consumption not knowing the realization of the state of the world, and all income over and above that level of private consumption becomes contributions to the public good. In a symmetric equilibrium, $x^i = x$ for any $i \in N$. This implies $g_s^i = m_s - x = G_S/n$ for any $i \in N$ and $s \in S$.

Proposition 4 *The condition that determines the interior symmetric equilibrium provisions of the public good and individual countries' consumption for Game 3 is*

$$u'(x) = \sum_{s \in \mathcal{S}} p_s v'(nm_s - nx). \tag{11}$$

This equilibrium exists if $u'(m_1) < \sum_{s \in \mathcal{S}} p_s v'(n(m_s - m_1))$ *and* $u'(0) > \sum_{s \in \mathcal{S}} p_s v'(nm_s)$*. When this symmetric equilibrium exists, it is the unique interior equilibrium.*

Proof. Consider first existence of interior symmetric equilibrium. Let G_1 be the quantity of the public good in state s. For interior symmetric equilibrium it must hold that $g_s^i = \frac{G_1}{n} + (m_s - m_1)$ for all $i = 1, ..., n$ and $G_s = G_1 + n(m_s - m_1)$. Using $x^i = m_1 - \frac{G_1}{n}$, the first-order condition can be written as

$$u'(m_1 - \frac{G_1}{n}) = \sum_{s \in \mathcal{S}} p_s v'(G_1 + n(m_s - m_1)). \tag{12}$$

The left-hand side of (6) is a continuous and strictly monotonically increasing function in $G_1 \in [0, nm_1]$. The right-hand side of (6) is a continuous and strictly monotonically decreasing function in $G_1 \in [0, nm_1]$. This equation has one and only one solution for interior $G_1 \in (0, nm_1)$ if $u'(m_1) < \sum_{s \in \mathcal{S}} p_s v'(n(m_s - m_1))$ (left-hand side smaller than right hand side at $G_1 = 0$) and $u'(0) > \sum_{s \in \mathcal{S}} p_s v'(nm_s)$ (left-hand side larger than right hand side at $G_1 = nm_1$). It remains to show that an interior equilibrium, if it exists, has to be symmetric. Suppose $G_1 \in (0, nm_1)$ is the equilibrium quantity of the public good in state $s = 1$. Note that, by construction of Game 3, country i's choice of x^i is the same for all states s. Suppose now $x^i > x^{i'}$ for some i and i'. Then $g_s^i < g_s^{i'}$ for all $s \in S$. This implies

$$\sum_{s \in \mathcal{S}} p_s v'(G_s) = u'(x^i) < u'(x^{i'}) = \sum_{s \in \mathcal{S}} p_s v'(G_s)$$

which is a contradiction. This completes the proof. ■

The proof of existence and uniqueness uses again that global income risk is the same for all countries, which reduces the dimensionality of the problem.

The existence condition requires that the marginal utility of the first unit of the private good is sufficiently high to prevent countries to expend all endowment on the public good, and that the marginal utility of private good is sufficiently declining to make some contribution to the public good worthwhile for them.

In analogy to Proposition 3, we can ask what is the implication of income uncertainty for the Nash equilibrium contributions to the public good. All utility risk is attributed to the public good benefit component. Compared to a situation without income risk, it is this utility component that becomes risky and absorbs all the risk. This motivates the following proposition, which holds under prudence.

Proposition 5 *Let $v''' > 0$ (prudence). In Game 3 systematic income risk increases the expected equilibrium provision to the public good compared to certain income.*

Proof. Let x be the symmetric equilibrium choice of private consumption in the equilibrium with income uncertainty and $\bar{x}$ be the private consumption in the absence of income risk. That is,

$$u'(x) = \sum_{s \in \mathcal{S}} p_s v'(nm_s - nx) \tag{13}$$

and

$$u'(\bar{x}) = v'(n\bar{m} - n\bar{x}).$$

Now, by Jensen's inequality it holds that if v' is convex, i.e., if $v''' > 0$ (prudence),

$$\sum_{s \in \mathcal{S}} p_s v'(nm_s - n\bar{x}) > v'(n\bar{m} - n\bar{x}) = u'(\bar{x}).$$

So, starting from $\bar{x}$, in order to fulfill (13), it is needed that u' goes up (lower x) and $\sum_{s \in \mathcal{S}} p_s v'(m_s - x)$ must go down, with also happens by decrease in x. So, the equilibrium private consumption is lower than $\bar{x}$ if income is uncertain in Game 3. A lower x corresponds to a higher expected contribution to the public good. This concludes the proof. ■

Proposition 5 is the mirror image to Proposition 3. If all risk that comes with risky national incomes is completely absorbed by one type of consumption, then this implies that the country (and so all countries) will shift more resources into this activity if the respective utility component (u or v respectively) is characterized by prudence. In order to facilitate this shift of resources, they have to cut back on the activity that is not risky.

Strict proportionality Both the case with fixed private consumption and with fixed contributions to the public good are extreme cases. An intermediate case is one in which changes in national incomes must be absorbed both by private consumption and public good contributions, but where a fine-tuning is not feasible, and where each country has to choose what share of the income the country will expend on private consumption, and which remaining share it will expend on contributions to the public good. If each country decides which share γ^i of national income to contribute to the public good in the different states, then the investment irreversibilities automatically determine the share of a country's national income which is privately consumed, for all possible states. Each of the n contributors maximizes

$$\max_{\gamma^i} \sum_{s=1}^{S} p_s u((1-\gamma^i)m_s) + \sum_{s=1}^{S} p_s v(\gamma^i m_s + \sum_{j \neq i} \gamma^j m_s). \tag{14}$$

In a symmetric equilibrium, $\gamma^i = \gamma$ for any $i \in N$. The following describes the contribution equilibrium:

Proposition 6 *The condition that determines the symmetric equilibrium provisions of the public good and individual countries' consumption for Game 4 is*

$$\sum_{s=1}^{S} p_s m_s u'((1-\gamma)m_s) = \sum_{s=1}^{S} p_s m_s v'(n\gamma m_s). \tag{15}$$

This equilibrium is unique if it exists and boundary conditions for existence of this equilibrium are: $\sum_{s=1}^{S} p_s m_s u'(m_s) < \bar{m}v'(0)$ *and* $\bar{m}u'(0) > \sum_{s=1}^{S} p_s m_s v'(nm_s)$*. When this symmetric equilibrium exists, it is the unique interior equilibrium.*

Proof. The first-order condition for optimal γ^i for country i is

$$\sum_{s=1}^{S} p_s m_s u'((1-\gamma^i)m_s) = \sum_{s=1}^{S} p_s m_s v'(\gamma^i m_s + \sum_{j \neq i} \gamma^j m_s) \tag{16}$$

This defines a system of n equations. Note that a solution to this system with $\gamma^i < \gamma^{i'}$ for some i and i' can be ruled out: suppose that such a solution exists. Then $\sum_{s=1}^{S} p_s m_s u'((1-\gamma^i)m_s) < \sum_{s=1}^{S} p_s m_s u'((1-\gamma^{i'})m_s)$ and $\sum_{s=1}^{S} p_s m_s v'(\gamma^i m_s + \sum_{j \neq i} \gamma^j m_s) > \sum_{s=1}^{S} p_s m_s v'(\gamma^{i'} m_s + \sum_{j \neq i'} \gamma^j m_s)$, and this contradicts that both γ^i and $\gamma^{i'}$ fulfill (16). Hence, $\gamma^i = \gamma^j \equiv \gamma$ in any candidate equilibrium, such that (16) can be written as

$$\sum_{s=1}^{S} p_s m_s u'((1-\gamma)m_s) = \sum_{s=1}^{S} p_s m_s v'(n\gamma m_s). \tag{17}$$

The left-hand side of (16) is continuously and strictly monotonically increasing in γ for $\gamma \in [0,1]$ from $\sum_{s=1}^{S} p_s m_s u'(m_s)$ to $\bar{m}u'(0)$, and the right-hand side of (17) is continuously and strictly monotonically decreasing from $\bar{m}v'(0)$ to $\sum_{s=1}^{S} p_s m_s v'(nm_s)$. Hence, a sufficient condition for existence and uniqueness of a γ that solves (17) is $\sum_{s=1}^{S} p_s m_s u'(m_s) < \bar{m}v'(0)$ and $\bar{m}u'(0) > \sum_{s=1}^{S} p_s m_s v'(nm_s)$. ■

Proportional contributions allow the countries to place some risk on both types of consumption. This may or may not increase the overall equilibrium contributions to the public good. But in comparison to fixed contributions or fixed consumption quantities, this equilibrium allocates risk in a more balanced way. And, if the two types of consumption affect utility similarly, then the choice of a quota can reproduce the equilibrium in which players make state-contingent contributions, as we show in the following section.

In analogy to Propositions 3 and 5, we can ask what is the implication of income uncertainty for the Nash equilibrium contributions to the public good. Unlike the previous cases, in Game 4 systematic income risk may increase or decrease the expected equilibrium provision to the public good compared to certain income, even when both u and v display prudence, regardless of whether u is "more" or "less" concave than v. The following example illustrates this:

Example 1: C*onstant Relative Risk-Aversion utility. Let*

$$u(x) = \frac{x^{1-\eta_1} - 1}{1 - \eta_1} \text{ and } v(y) = \frac{y^{1-\eta_2} - 1}{1 - \eta_2}, \tag{18}$$

where the relative risk-aversion coefficient for u *(respectively,* v*) is* η_1 *(*η_2*), with* $\eta_1, \eta_2 > 0$*. Let the number of agents be* 2*. Let the uncertain income case be characterized by* $p_1 = 1/3$, $m_1 = 2$, $p_2 = 2/3$, $m_2 = 5$*. Let the certain income be characterized by* $\overline{m} = p_1 m_1 + p_2 m_2 = 4$*. Let* γ *be the symmetric equilibrium choice of share of income devoted to the public good in the equilibrium with income uncertainty and* $\overline{\gamma}$ *be the chosen share in the absence of income risk. The following table summarizes the comparison between* γ *and* $\overline{\gamma}$ *for several values of* η_1 *and* η_2*.*

Table 1: proportional contributions to the public good with and without income uncertainty

	γ	$\overline{\gamma}$
1) $\eta_1 = 0.3$, $\eta_2 = 0.3$	0.3333	0.3333
2) $\eta_1 = 0.4$, $\eta_2 = 0.3$	0.4016	0.4009
3) $\eta_1 = 0.3$, $\eta_2 = 0.4$	0.2766	0.2772
4) $\eta_1 = 0.8$, $\eta_2 = 0.4$	0.4990	0.5000
5) $\eta_1 = 0.4$, $\eta_2 = 0.8$	0.2213	0.2207

As Table 1 shows, income uncertainty may leave contributions unchanged, as in line 1), increase them, as in lines 2) and 5), or decrease them, as in lines 3) and 4). Note as well that increases and decreases in contributions can occur both when u *is "more" concave than* v *(*$\eta_1 > \eta_2$*) and when* u *is less concave than* v *(*$\eta_1 < \eta_2$*).* ∎

4 Comparisons

For the same type of aggregate income uncertainty, the equilibria in the four games generally differ. We first compare the expected quantity of the public good.

4.1 Expected public good amounts

Each country's consumption pattern changes due to the outcome of income uncertainty. The differences in shape of u and v affect how uncertainty affects the allocation choices in each of the four equilibria, as analyzed in the previous section. This makes a full comparison for the four equilibria for general u and v cumbersome and dependent on the details of the income distribution. Some insight into the structural properties can be gained, however, if the problem is drastically simplified, assuming that $u = v$.

Proposition 7 *Let $u = v$. (i) For the benchmark state-contingent case (Game 1), the equilibrium allocation has*

$$x_s = G_s = \frac{nm_s}{n+1}. \tag{19}$$

(ii) For Game 2, the equilibrium quantity of the public good is smaller than the expected equilibrium public good amount for the state-contingent case if u is characterized by prudence (i.e., $u'''(x) > 0$). (iii) For Game 3 the equilibrium quantity of private consumption is smaller than the expected equilibrium private consumption for the state-contingent case if u is characterized by prudence (i.e., $u'''(x) > 0$). Therefore, the expected equilibrium public good amount is larger in Game 3 than in Game 1 (iv) For Game 4 the first-order condition reproduces the equilibrium quantities of Game 1.

Proof. (i) For $u = v$, the first-order condition (4) that characterizes the symmetric equilibrium in Game 1 reduces to

$$m_s - \frac{1}{n}G_s = G_s \ \forall s \in \mathcal{S}, \tag{20}$$

or

$$G_s = \frac{n}{n+1}m_s. \tag{21}$$

As $x_s = m_s - G_s/n$ in the symmetric equilibrium, we also obtain $x_s = G_s$ $\forall s \in \mathcal{S}$. For the purpose of further comparisons, we denote these equilibrium levels as G_s^I and x_s^I, with $G_s^I = x_s^I = m_s \cdot n/(n+1)$.

(ii) Let the equilibrium public good amount in Game 2 be G^{II}, and assume by contradiction that $G^{II} > \sum_{s\in\mathcal{S}} p_s G_s^I$. The first-order condition that characterizes equilibrium in Game 2 is

$$u'(G^{II}) = \sum_{s\in\mathcal{S}} p_s u'(x_s^{II}). \tag{22}$$

Using Jensen's inequality, for $u''' > 0$ this implies

$$u'(G^{II}) > u'(\sum_{s\in\mathcal{S}} p_s x_s^{II}). \tag{23}$$

Note that by the budget constraint $G^{II} > \sum_{s\in\mathcal{S}} p_s G_s^I$ if and only if $\sum_{s\in\mathcal{S}} p_s x_s^{II} < \sum_{s\in\mathcal{S}} p_s x_s^I$. Hence,

$$u'(\sum_{s\in\mathcal{S}} p_s x_s^{II}) > u'\left(\sum_{s\in\mathcal{S}} p_s x_s^I\right). \tag{24}$$

Now, due to $u = v$, it holds that $x_s^I = G_s^I$. Hence,

$$u'\left(\sum_{s\in\mathcal{S}} p_s x_s^I\right) = u'\left(\sum_{s\in\mathcal{S}} p_s G_s^I\right). \tag{25}$$

Comparing the beginning and the end of this chain of inequalities we obtain

$$u'(G^{II}) > u'(\sum_{s\in\mathcal{S}} p_s G_s^I). \tag{26}$$

This implies $G^{II} < \sum_{s\in\mathcal{S}} p_s G_s^I$ and this contradicts the starting assumption that $G^{II} > \sum_{s\in\mathcal{S}} p_s G_s^I$.

(iii) Let the equilibrium private consumption amount in Game 3 be x^{III}, and assume by contradiction that $x^{III} > \sum_{s\in\mathcal{S}} p_s x_s^I$. We proceed with a similar chain of inequalities as we did for part (ii). The first-order condition characterizing the equilibrium with fixed private consumption in Game 3 is:

$$u'(x^{III}) = \sum_{s\in\mathcal{S}} p_s u'(G_s^{III}). \tag{27}$$

By Jensen's inequality,

$$u'(x^{III}) = \sum_{s\in\mathcal{S}} p_s u'(G_s^{III}) > u'(\sum_{s\in\mathcal{S}} p_s G_s^{III}). \tag{28}$$

Note that by the budget constraint $x^{III} > \sum_{s\in\mathcal{S}} p_s x_s^I$ if and only if $\sum_{s\in\mathcal{S}} p_s G_s^{III} < \sum_{s\in\mathcal{S}} p_s G_s^I$. Therefore, by concavity of u we have

$$u'(\sum_{s\in\mathcal{S}} p_s G_s^{III}) > u'\sum_{s\in\mathcal{S}}(p_s G_s^I). \tag{29}$$

Now we use $x_s^I = G_s^I$ for $u = v$ and obtain

$$u'\left(\sum_{s\in\mathcal{S}} p_s G_s^I\right) = u'\left(\sum_{s\in\mathcal{S}} p_s x_s^I\right). \tag{30}$$

Overall, this chain of inequalities leads to $u'(x^{III}) > u'(\sum_{s\in\mathcal{S}} p_s x_s^I)$, which, by concavity of u, implies $x^{III} < \sum_{s\in\mathcal{S}} p_s x_s^I$, which contradicts the assumption. This contradiction completes the proof.

(iv) The claim follows immediately from making use of $u = v$ in (15), because $\gamma = 1/(n+1)$ is such that

$$(1-\gamma)m_s = n\gamma m_s \ \forall s \in \mathcal{S} \tag{31}$$

and so (15) is satisfied. Furthermore, $(1-\gamma)m_s = x_s^I = G_S^I = \frac{n}{n+1}m_s$, if we use $\gamma = 1/(n+1)$. ■

Proposition 7 sorts the games in terms of the expected contribution to the public good. Games 1 and 4 yield precisely the same equilibrium allocation. This, however, is a knife-edge result: Game 4 induces constant shares, and such constant shares are optimal under very specific conditions, $u = v$ being sufficiently specific in this respect. As we compare Games 1–3, the expected contribution is largest in Game 3, followed by Game 1 and then Game 2. Intuitively, the global income risk is absorbed differently in the four games. Games 2 and 3 are more extreme in how the income risk is absorbed than in Game 1. In Game 2, all income risk is absorbed by private consumption (whereas g_i is fixed); in Game 3, all income risk is absorbed by public good provision. As we compare Game 1 to Game 2, less risk is shifted to the private consumption sector in the latter. Prudent players will therefore allocate more resources to this sector. As we compare Game 1 to Game 3, less risk is shifted to the public good sector in the latter. Prudent players will therefore allocate more resources to this sector.

4.2 Ex-ante welfare

One can divide the schemes that we are considering into restrictive (Games 2, 3, and 4) and non-restrictive (scheme 1). In other words, in Game 1 countries are allowed to adjust—independently in each state—their private consumptions and their contributions to the public good. In contrast, in Games 2, 3, and 4, countries are either constrained to a fixed level of public good contributions, a fixed level of private consumption, or an across-states constant proportional division of income between private consumption and public good contribution. Not surprisingly, one can show that each of the restrictive schemes may dominate the other, depending on the specific utility function. More surprisingly, it turns out that having a restrictive scheme may end up benefiting countries with respect to the non-restrictive scheme. On the one hand, the non-restrictive scheme does always better in an individualistic problem, simply because a constrained maximization problem cannot give a larger utility than an unconstrained one. On the other hand, when multiple agents are contributing to a public good, it is possible that a restriction imposed to an individual's choice may result into a larger public good contribution in some states, and in turns this may end up alleviating the free-rider problem and lead to a Pareto improvement, with respect to a non-restrictive scheme.

Part (iii) of Proposition 7 already hints at this result, as it shows that the scheme with fixed national consumption leads to larger expected public good provision if $u = v$ and $v''' > 0$. But this scheme is the least realistic of those that we consider. And Proposition 7 says nothing about welfare. We therefore turn to strict proportionality (Game 4). As it happens for the public good level, a full comparison of welfare between strict proportionality and state-contingent provision for general u and v is cumbersome, and the result will depend on the details of the income distribution. Therefore, we illustrate our point that restrictions caused by technological irreversibilities may be welfare-improving assuming specific functional forms for utility and comparing the non-restrictive scheme with state-contingent public good provision against the restrictive scheme with proportional contributions. In particular, we consider the following pair of utility functions:

$$u(x) = \frac{x^{1-\eta} - 1}{1-\eta} \text{ and } v(G) = G. \tag{32}$$

Under these assumptions, we obtain the following result.[11]

Proposition 8 *Let $u = u(x) = \frac{x^{1-\eta}-1}{1-\eta}$, with $\eta \in (0,1)$, and $v(G) = G$. Ex-ante expected utility in the equilibrium of the game with strict proportionality (Game 4) is larger than that of the state-contingent game (Game 1) if and only if*

$$n > \frac{1}{1-\eta}. \tag{33}$$

The proof of Proposition 8 is in the Appendix. Note that Proposition 8 implies that if $n = 1$, then $EU^P < EU^{SC}$. This intuitive results occurs because the only force at play is the loss of flexibility in the individual maximization problem. But if $n > 1$, then EU^P can be larger than EU^{SC}. Furthermore, for any fixed $\eta \in (0,1)$ there exists an $\bar{n} < \infty$ such that $EU^P > EU^{SC}$ if the number of countries n is at least as large as $\bar{n}$. This result accords with the intuition at the beginning of this section: restricting individual choice for everybody may end up alleviating the free-rider problem, which is especially severe if the number of potential contributors is large.

5 Conclusion

In a closely interconnected world economy, the possibility of events such as world financial crisis, pandemics, or disruptive technological developments causes global systematic and correlated income risk for countries. The following questions emerge: how do countries address such uncertainties when making contributions to climate protection, and how do their equilibrium contributions depend on irreversibilities in their investment choices? Choices on the energy mix and the carbon lock-in it implies, for instance, limit the scope for states to freely react and adjust their climate protection efforts to high- or low-income outcomes. It is also possible that these investment decisions fix climate protection contributions and expenditures for national concerns as more or less constant shares to each other.

This paper determines the different Nash provision equilibria in the presence of global income risks, and compares how the different constraints on ex-post adjustment to the risky income outcomes affect the equilibrium contribution outcomes. If ex-ante choices of climate spending cannot adjust to income shocks, such risk will, under plausible conditions increase expected spending for purely national purposes and lead to lower climate contributions. The analysis takes

[11] The result is given for linear v for tractability. By continuity, linearity is not necessary.

the fully flexible allocation between contributions to international climate protection and purely national concerns as a benchmark. It assumes that countries' preferences are characterized by prudence. It then shows that technological commitment (such as carbon lock-in) tends to lead to less climate protection in expectation, while a specification of risk-independent expenditures for national interests tends to lead to higher expenditures for climate protection in expectation terms.

The paper offers several further comparisons, some of which depend on the details of the problem at hand, but all of them highlight that the question of whether and how a country can adjust its major expenditure types to global income shocks is an important aspect for the climate policy outcome. It is precisely the commitment to climate policy to remain unaffected by income shocks that may lead to lower climate policy efforts than would be made in expectation without this growth independence.

Of course, such results from a strictly non-cooperative equilibrium analysis are not to be understood one-to-one as predictions for climate policy. In climate policy, countries negotiate with each other and the results are not strictly non-cooperative. However, as often emphasised in other contexts, the prediction of the non-cooperative Nash equilibrium outcome is also relevant for climate negotiations insofar as it (co-)determines the threat point of such negotiations, and it is the default in case such negotiations fail.

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Appendix A - Proofs

Proof of Proposition 8. For state-contingent provision, by (4) we have

$$u'(m_s - g_s) = 1,$$

which results in $g_s = m_s - 1$. Thus, $\min_s\{m_s\} \geq 1$ ensures an interior solution in which each agent privately consumes $x_s = 1$ in each state. Therefore, the equilibrium utility from private consumption equals zero, and ex-ante expected utility in the equilibrium of the state-contingent provision game is

$$EU^{SC} = n\left(\sum_{s\in S} p_s m_s - 1\right).$$

In the proportional contribution scheme, (15) yields

$$\sum_{s\in S} p_s m_s\left(-u'\left(m_s(1-\gamma)\right)+1\right) = 0,$$

which implies that the optimal level of γ, denoted as γ^*, solves

$$\sum_{s\in S} p_s (m_s)^{1-\eta} = (1-\gamma^*)^\eta \sum_{s\in S} p_s m_s. \tag{34}$$

Using this result, ex-ante expected utility is in the equilibrium of the proportional provision game is

$$\begin{aligned} EU^P &= \sum_{s\in S} p_s\left(\tfrac{((1-\gamma^*)m_s)^{1-\eta}-1}{1-\eta} + n\gamma^* m_s\right) \\ &= \frac{(1-\gamma^*)^{1-\eta}\sum_{s\in S} p_s(m_s)^{1-\eta}}{1-\eta} + n\gamma^*\sum_{s\in S} p_s m_s - \tfrac{1}{1-\eta} \\ &= \frac{((1-\gamma^*)+(1-\eta)n\gamma^*)\sum_{s\in S} p_s m_s}{1-\eta} - \tfrac{1}{1-\eta}. \qquad \text{(by (34))} \end{aligned}$$

Therefore, simple algebra implies that $EU^P > EU^{SC}$ is equivalent to

$$(1-(1-\eta)n)\left((1-\gamma^*)\sum_{s\in S} p_s m_s - 1\right) > 0. \tag{35}$$

We now show $(1-\gamma^*)\sum_{s\in S} p_s m_s < 1$. We have

$$\begin{aligned} (1-\gamma^*)\sum_{s\in S} p_s m_s &= \left(\frac{\sum_{s\in S} p_s(m_s)^{1-\eta}}{\sum_{s\in S} p_s m_s}\right)^{\frac{1}{\eta}} \sum_{s\in S} p_s m_s \qquad \text{(by (34))} \\ &= \left(\sum_{s\in S} p_s (m_s)^{1-\eta}\right)^{\frac{1}{\eta}} \left(\sum_{s\in S} p_s m_s\right)^{1-\frac{1}{\eta}} \\ &= \left(\sum_{s\in S} p_s (m_s)^{1-\eta}\right)^{\frac{1}{\eta}} \left(\sum_{s\in S} p_s m_s\right)^{\frac{\eta-1}{\eta}} \\ &= \left(\frac{\sum_{s\in S} p_s(m_s)^{1-\eta}}{\left(\sum_{s\in S} p_s m_s\right)^{1-\eta}}\right)^{\frac{1}{\eta}} \\ &< 1, \end{aligned}$$

where the last inequality follows because Jensen's inequality yields

$$\sum\nolimits_{s\in S} p_s (m_s)^{1-\eta} < \left(\sum\nolimits_{s\in S} p_s m_s\right)^{1-\eta}.$$

Therefore, we have established $(1-\gamma^*)\sum\nolimits_{s\in S} p_s m_s < 1$ so (35) gives us:

$$EU^P > EU^{SC} \Leftrightarrow n > (1-\eta)^{-1},$$

as we wanted to show. ∎

Appendix B - Additional Results

This Appendix contains additional results that are more technical in nature. We begin with the analysis of the implications of income uncertainty for the expected level of public good provision in Game 1, in analogy to what we did in Proposition 3 for Game 2 and in Proposition 5 for Game 3. We have two cases. First, we consider the case of systematic income risk where there are many possible states indexed by s, with income m_s in state s, and each state occurs with probability p_s. We define G_s as the equilibrium level of the total public good, as characterized in (4). We compare this to the case where is no income uncertainty: there is only one state, with income level $\bar{m} = \sum\limits_s p_s m_s$. We define $(\bar{x}, \bar{G})$ the level of the private and public good at the Nash equilibrium. The following proposition shows that the sign of the comparison can be characterized by the function $\phi(x) \equiv \left[u^{'}\right]^{-1}(v^{'}(x))$.

Proposition 9 *In Game 1 (full ex-post flexibility) systematic income risk increases the equilibrium provision to the public good compared to certain income if $\phi(x)$ is strictly concave. If $\phi(x)$ is strictly convex, then systematic income risk decreases the equilibrium provision to the public good compared to certain income. And if $\phi(x)$ is linear, then systematic income risk does not affect the equilibrium provision of the public good.*

Proof. We prove only the case of $\phi(x)$ strictly concave, as the other cases are similar. Formally, we want to prove that $\bar{G} < \sum\limits_s p_s G_s$. Without income uncertainty, standard first-order condition reasoning, along with symmetry, making use of the definition of $\phi(x)$ yields

$$u^{'}(\bar{m} - \frac{\bar{G}}{n}) = v^{'}(\bar{G})$$

implies

$$\bar{m} = \frac{\bar{G}}{n} + \phi(\bar{G}). \tag{36}$$

With income uncertainty, (4) implies that in state s we have

$$m_s = \frac{G_s}{n} + \phi(G_s),$$

and multiplying both sides of the above-displayed equation by p_s and adding over states we obtain

$$\sum_s p_s m_s = \frac{1}{n}\sum_s p_s G_s + \sum_s p_s \phi(G_s). \tag{37}$$

As $\sum_s p_s m_s = \bar{m}$, (36) and (37) imply

$$\frac{\bar{G}}{n} + \phi(\bar{G}) = \frac{1}{n}\sum_s p_s G_s + \sum_s p_s \phi(G_s). \tag{38}$$

If ϕ is strictly concave, then Jensen's inequality gives us $\sum_s p_s \phi(G_s) < \phi(\sum_s p_s G_s)$. Therefore (38) implies

$$\frac{\bar{G}}{n} + \phi(\bar{G}) < \frac{1}{n}\sum_s p_s G_s + \phi(\sum_s p_s G_s). \tag{39}$$

Since $\phi'(x) = v''(x)/u''(\phi(x)) > 0$ by concavity of u and v, the function $\frac{x}{n} + \phi(x)$ is increasing in x; therefore, (39) gives $\bar{G} < \sum_s p_s G_s$ as we wanted to show. ■

Now, we consider the implications of income uncertainty for the expected level of public good provision in Game 4, in analogy to what we did in Proposition 3 for Game 2 and in Proposition 5 for Game 3. Despite the variety of results illustrated in Example 1, it is possible to provide a sufficient condition for the sign of the comparison of public good provision with or without income uncertainty. Let γ be the symmetric equilibrium choice of share of income devoted to the public good in the equilibrium with income uncertainty, and define

$$\psi(m) \equiv m\left(-u'\left((1-\gamma)m\right) + v'\left(n\gamma m\right)\right).$$

We then obtain the following result.

Proposition 10 *Suppose $\psi(m)$ is a convex (concave) function of m when $m \in [m_1, m_S]$. Then increased income uncertainty decreases (increases) expected public good provision.*

Proof. The first-order condition with income risk is

$$0 = \sum_{s=1}^{S} p_s m_s \left(-u'((1-\gamma)m_s) + \sum_{s=1}^{S} p_s m_s v'(n\gamma m_s)\right). \tag{40}$$

Using convexity of $\psi(m)$ in m, by (40) and Jensen's inequality we obtain

$$\begin{aligned}
0 &= \sum_{s=1}^{S} p_s \left(m_s \left(-u'((1-\gamma)m_s) + \sum_{s=1}^{S} p_s m_s v'(n\gamma m_s) \right) \right) \\
&> \left(\sum_{s=1}^{S} p_s m_s \right) \left(-u'((1-\gamma) \sum_{s=1}^{S} p_s m_s) + \sum_{s=1}^{S} p_s m_s v'(n\gamma \sum_{s=1}^{S} p_s m_s) \right) \\
&= \overline{m} \left(-u' \left((1-\gamma) \overline{m} \right) + v' \left(n\gamma \overline{m} \right) \right),
\end{aligned}$$

which gives

$$u' \left((1-\gamma) \overline{m} \right) > v' \left(n\gamma \, \overline{m} \right).$$

As both u' and v' are decreasing and the first-order condition without income risk is

$$u'((1-\overline{\gamma})\overline{m}) = v'(n\overline{\gamma}\, \overline{m}), \tag{41}$$

we see that $\gamma > \overline{\gamma}$. The case of $\psi(m)$ concave in m is similar and here omitted. ■

One can verify that $\psi(m) = 0 \ \forall m$ for the values in line 1) in Table 1, for which income uncertainty has no effect, and that $\psi(m)$ is convex for the values in lines 2) and 5), so that income uncertainty increases provision, and that $\psi(m)$ is concave for the values in lines 3) and 4), so that income uncertainty decreases provision, with all results in line with Proposition 10.